EXCELLENCIA
INTERNATIONAL MULTI-DISCIPLINARY JOURNAL OF EDUCATION

Volume 02, Issue 04, 2024 ISSN (E): 2994-9521

Representation of the Middle Line of a Trapeziod Through the Sides

Ghafforov Husayn Aliyarovich
Tashkent State Agrarian University
Teacher of the Department of Mathematics and its Teaching Methodology, Faculty of Mathematics and Informatics, Termiz State Pedagogical Institute.

Jurayev Ilhom Ruziboyevich
Head of the Department of Mathematics and its Teaching Methodology, Faculty of Mathematics and Informatics, Termiz State Pedagogical Institute.
avazovazizbek@internet.ru

Khudoyorova Zhangil Ashurovna
Termiz State Pedagogical Institute, Faculty of Mathematics and Informatics 60110600- 3rd level student of Mathematics and Informatics.

Abstract

This article describes the method of expressing the second midline of a trapezoid using the property of the median of a tringle in terms of the sides the trapezium.
Key words: triangle median property, proof, sides of a trapezoid, equal sections.

Introduction

We all know that there is a formula for expressing the median of a triangle by its sides. It looks like this:

$$m_c = \frac{1}{2} * \sqrt{2a^2 + 2b^2 - c^2} \qquad (1)$$

We present the proof of formula (1). For this, we will first use Stuart's theorem and study its proof.

$$BD^2 = \frac{BC^2 \cdot AD}{AC} + \frac{AB^2 \cdot DC}{AC} - AD \cdot DC \qquad (2)$$

Let's get acquainted with the proof of Stuart's theorem:

Proof: Let ABC be an arbitrary triangle. We make an arbitrary cut from one end of the triangle to the other. We apply the theorem of cosines to the 2 resulting triangles. **(Figure 1)**

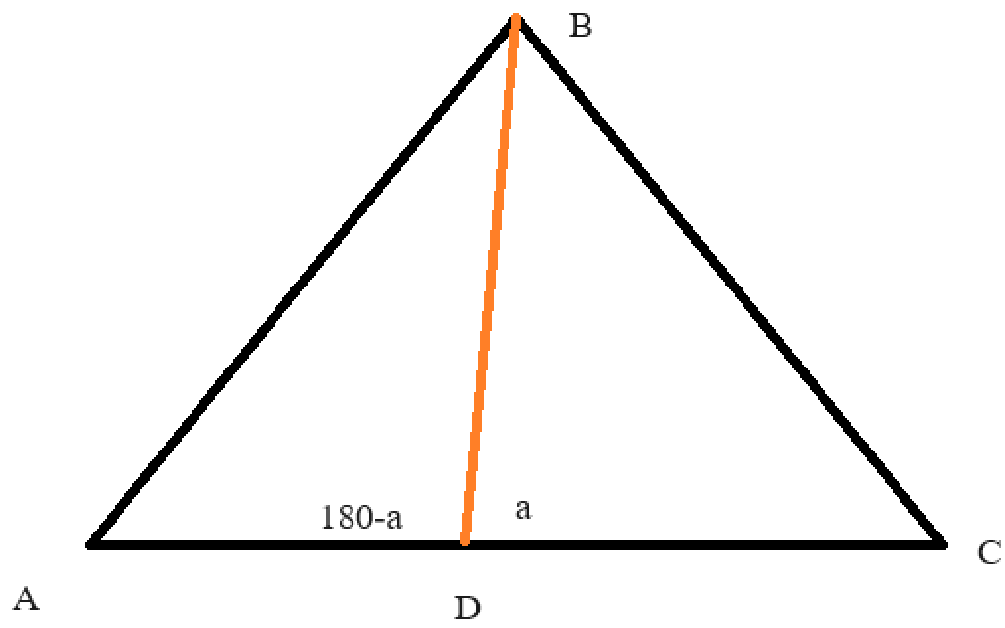


Figure 1

$$AB^2 = AD^2 + BD^2 - 2AD \cdot BD \cdot \cos(\pi - \varphi)$$

$$BC^2 = DC^2 + BD^2 - 2DC \cdot BD \cdot \cos\varphi$$

Of both $\cos\varphi$ find and equate the expressions:

$$\cos(\pi - \varphi) = -\cos\varphi$$

$$\cos\varphi = \frac{BD^2 + DC^2 - BC^2}{2BD \cdot DC}$$

$$\cos\varphi = \frac{AD^2 + BD^2 - AB^2}{-2BD \cdot AD}$$

$$\frac{BD^2 + DC^2 - BC^2}{2BD \cdot DC} = \frac{AD^2 + BD^2 - AB^2}{-2BD \cdot AD}$$

$$-\left(BD^2 + DC^2 - BC^2\right) \cdot AD = \left(AD^2 + BD^2 - AB^2\right) \cdot DC$$

$$AD^2 \cdot DC + BD^2 \cdot DC - AB^2 \cdot DC = -BD^2 \cdot AD - DC^2 \cdot AD + BC^2 \cdot AD$$

$$AD^2 \cdot DC + BD^2 \cdot DC + BD^2 \cdot AD + DC^2 \cdot AD = BC^2 \cdot AD + AB^2 \cdot DC$$

$$AD\cdot DC\cdot(AD+DC)+BD^2\cdot(AD+DC)=BC^2\cdot AD+AB^2\cdot DC$$

$$\{AD+DC=AC\}$$

$$AD\cdot DC\cdot AC+BD^2\cdot AC=BC^2\cdot AD+AB^2\cdot DC$$

$$BD^2\cdot AC=BC^2\cdot AD+AB^2\cdot DC-AD\cdot DC\cdot AC$$

$$BD^2=\frac{BC^2\cdot AD}{AC}+\frac{AB^2\cdot DC}{AC}-AD\cdot DC$$

the theorem is proved.

We can also prove the formula for the median of an arbitrary triangle using Stuart's theorem. **(Figure 2)**

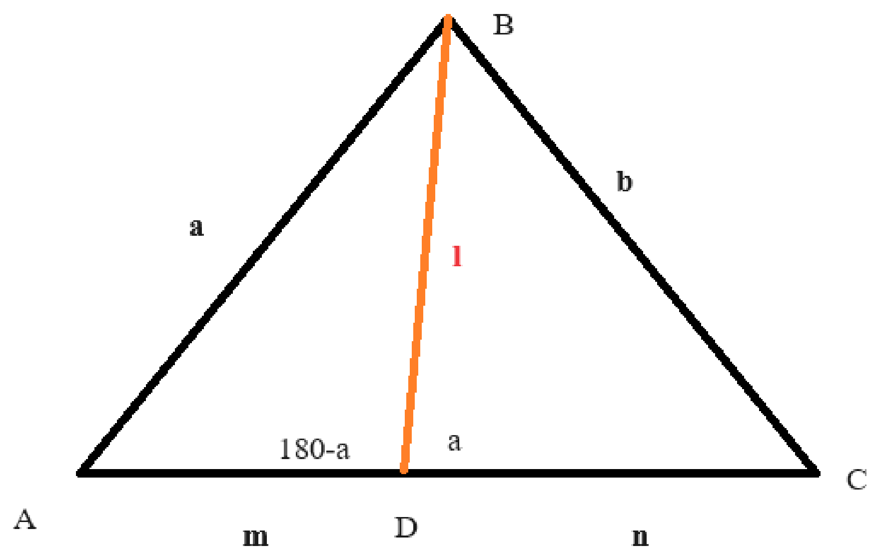


Figure 2

$$BD^2=\frac{BC^2\cdot AD}{AC}+\frac{AB^2\cdot DC}{AC}-AD\cdot DC$$

We write down Stuart's theorem according to our drawing.

$$l_c{}^2=\frac{a^2\cdot n}{c}+\frac{b^2\cdot m}{c}-m\cdot n$$

Let us consider the proof of this median formula.

$$m=\frac{c}{2} \qquad n=\frac{c}{2}$$

$$m_c=l_c$$

$$l_c^{\,2}=\frac{a^2\cdot c}{2\cdot c}+\frac{b^2\cdot c}{2\cdot c}-\frac{c}{2}\cdot\frac{c}{2}$$

$$m_c^{\,2}=\frac{a^2+b^2}{2}-\frac{c^2}{4}$$

$$m_c^{\,2}=\frac{2\cdot\left(a^2+b^2\right)-c^2}{4}$$

$$m_c=\frac{1}{2}\cdot\sqrt{2\cdot\left(a^2+b^2\right)-c^2}$$

$$m_c=\frac{1}{2}\cdot\sqrt{2\cdot\left(a^2+b^2\right)-c^2}$$

The median formula was proved.

Now, let us be given a trapezoid ABCD. **(Figure 3)**

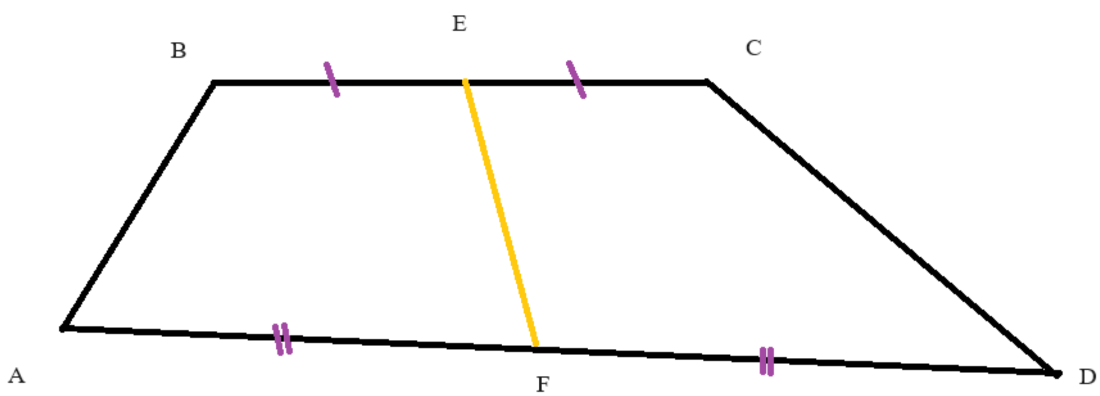
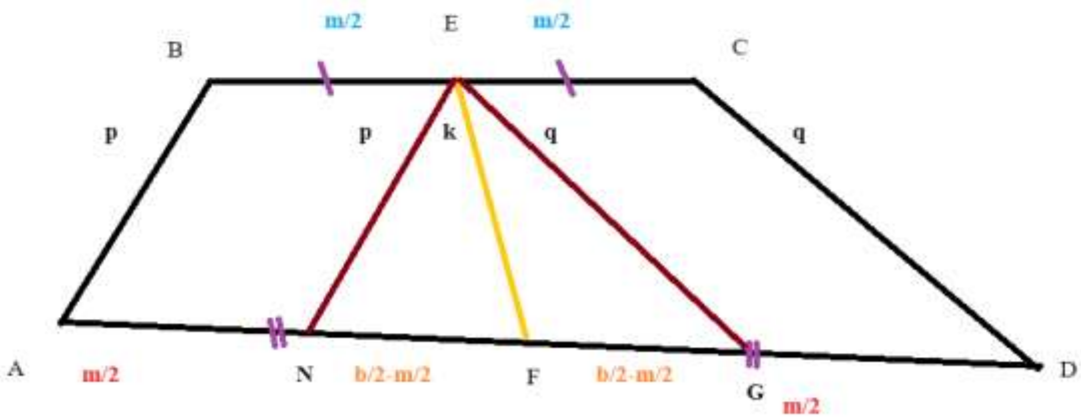


Figure 3

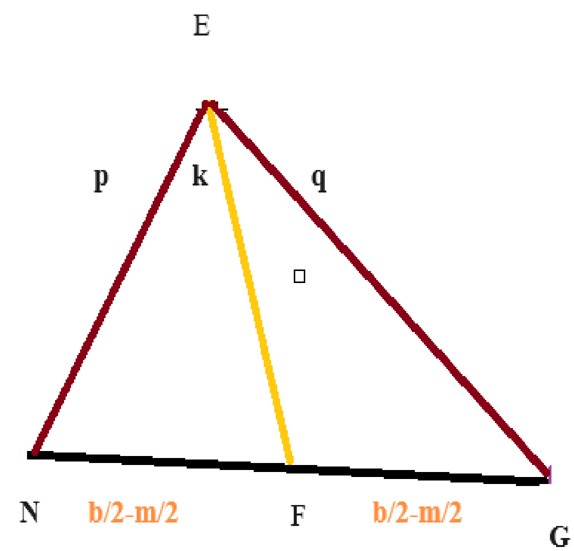
Here, k- is the section connecting the middle of the bases of the trapezoid ABCD, or we can take it as the second middle line of the trapezoid. BE=EC=AE=ND=m/2; AB=EF=p; EN=CD=q;

We draw sections EF and EG parallel to sides AB and CD and ending at point E. **(Figure 4).**



Future 4

Let's pay attention to the triangle ENG. **(Figure 5)**



Future 5

Based on the above formula for finding the median of a triangle,

$$l=\frac{1}{2}*\sqrt{2p^2+2q^2-(b-m)^2}=\frac{1}{2}*\sqrt{2p^2+2q^2-b^2-m^2+2mb}=$$

(3)

$$=\sqrt{\frac{1}{2}p^2+\frac{1}{2}q^2-\frac{1}{4}b^2-\frac{1}{4}m^2+\frac{1}{2}mb}$$

$$l = \sqrt{\frac{1}{2}p^2 + \frac{1}{2}q^2 - \frac{1}{4}b^2 - \frac{1}{4}m^2 + \frac{1}{2}mb}$$

REFERENCES:

1. Погорелов “Геометрия” Ўрта мактабнинг 7-11 синф ўқувчилари учун ўқув қўлланма. Тошкент: “Ўқитувчи”- 1990.
2. Nazarov H.H., X.O.Ochilova va boshqalar “Geometriydan masalalar to‘plami 1 qism” Toshkent “O‘qituvchi” nashriyoti tuzatilgan 1997 yil
3. B.Xaydarov, N.Toshtemirova, I.Asrorov “Geometriya umumiy o‘rta ta’lim maktablarining 7-sinflari uchun” Toshkent Respublika ta’lim markazi-2022