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THE IMPORTANCE AND ROLE OF MATHEMATICAL MODELING IN THE STUDY AND PREDICTION OF HYDRODYNAMIC PARAMETERS OF FILTRATION PROCESSES IN GAS FIELDS.

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Abstract:

The article presents methods for constructing mathematical models of non-stationary filtration processes that take into account the structure of gas layers in porous media. The well development time, pressure, gas parameters - viscosity, layer permeability, its dimensions, as well as the number of wells and their coordinates, well flow rate and initial porosity coefficient values are given, as well as the ability to view 2D pressure in the layer section and 3D graphs of the complete diffusion processes in the layer. In addition, the pressure drop in gas layer wells and the pressure-dependent change in porosity are also reflected. The necessary information is provided on the application of effective computational methods to solve the boundary problem. These methods serve to solve problems of non-stationary gas filtration processes.

Keywords: Filtration processes, mathematical models, layer pressure, finite difference method, software interface.

1. Introduction

The study and prediction of filtration processes in gas fields, taking into account changes in hydrodynamic parameters, is considered one of the important scientific and practical problems. Efficient management of these processes is crucial for maximizing the utilization of gas fields, conserving resources, and enhancing the economic efficiency of production processes. To achieve this goal, modern mathematical models and numerical simulations play a vital role in accurately analyzing gas field parameters and predicting their development under various hydrodynamic conditions. Such models not only enable precise analysis of gas movement but also improve economic efficiency and promote rational resource utilization. Through computational algorithms and visualization tools, these processes can be analyzed accurately and presented in a simplified manner[5].

Currently, significant attention is being paid to developing mathematical models, efficient numerical algorithms, and software solutions to address the challenges that arise when studying and predicting filtration processes in single-layer and multi-layer gas fields. This highlights the importance of creating mathematical models that account for changes in hydrodynamic parameters during filtration processes, analyze numerical results, and present them in visual, graphical, and animated formats through computer-based models. Improving computational algorithms remains one of the key tasks of the day. Furthermore, the development of mathematical models, efficient numerical algorithms, and software packages for non-stationary filtration processes in dynamically interconnected gas layers, where porosity coefficients vary depending on pressure, represents targeted scientific research.

To date, nearly 300 oil and gas fields have been discovered in our country. The application of modern technologies, rapid computations, and purposeful use of the latest computer technology capabilities are among the pressing issues in the extraction of these fields. Solving practical problems in the oil and gas industry requires systematic programs that involve numerical modeling and the use of computational algorithms on modern computers to enhance efficiency. In this process, applying quasi-linear methods alongside finite difference methods and the method of characteristics in each direction is essential for calculating the main indicators of gas fields while considering hydrodynamic parameters. Accounting for hydrodynamic parameters in calculations, enabling rapid computations, and predicting subsequent pressure changes in gas wells are regarded as priority issues from both economic and safety perspectives[8].

Improving the efficiency of oil and gas field operations holds significant importance in the oil and gas industry. In the development of this sector, mathematical modeling plays a crucial role in maintaining artificial pressure in oil or gas layers through water flooding, increasing layer productivity, and forecasting key performance indicators. Additionally, creating monitoring information systems for oil and gas extraction is vital.

It is well known that in oil fields, a drop in pressure within the layers is observed over time at certain stages. In such cases, to increase oil production, the pressure in the layers is artificially increased during oil field operations by using the "flooding" method. This method involves injecting water either from outside the oil contour or from within the contour. The external flooding method is applied to small-volume oil fields, where water is injected through wells located very close to the outer contour. The internal flooding method is used for oil layers with low permeability coefficients, where water is injected through partially water-filled oil wells or nearby water wells.

2. Materials and Methods

Mathematical modeling of such problems for two-layer oil fields with low-permeability layers that are dynamically interconnected is considered quite complex[9].

Mathematically, such processes in a two-layer porous medium are formulated as a "moving boundary oil-water problem," and the problem is expressed in the following boundary value form.

$$\frac{\partial}{\partial x}\left[\frac{k_1(x)h_1(x)}{\mu_o}\frac{\partial p_1}{\partial x}\right]-q_{1o}\delta(x-x_{io})=\beta_{1o}h_1(x)\frac{\partial p_1}{\partial t}-\frac{k_{\Pi}(x)h_1(x)}{\mu_o h_{\Pi}(x)}(p_2-p_1),\quad (1)$$

$$l_1<x<l;$$

$$\frac{\partial}{\partial x}\left[\frac{k_2(x)h_2(x)}{\mu_o}\frac{\partial p_2}{\partial x}\right]-q_{2o}\delta(x-x_{io})=\beta_{2o}h_2(x)\frac{\partial p_2}{\partial t}+\frac{k_{\Pi}(x)h_2(x)}{\mu_o h_{\Pi}(x,y)}(p_2-p_1),\quad (2)$$

$$l_2<x<l;$$

$$\frac{\partial}{\partial x}\left[\frac{k_2(x)h_2(x)}{\mu_w}\frac{\partial p_2}{\partial x}\right]+q_{2w}\delta(x-x_{iw})=\beta_{2w}h_2(x)\frac{\partial p_2}{\partial t},\quad 0<x<l_2.\quad (3)$$

Initial and boundary conditions:

$$p_1=p_2=p_н,\quad t=0,\quad (4)$$

$$\frac{\partial p_1}{\partial x}=0,\quad x=0;\quad \frac{\partial p_2}{\partial x}=0,\quad x=0;\quad (5)$$

$$p_1(x)\big|_o = p_1(x)\big|_w, \quad \frac{k_1}{\mu_o}\frac{\partial p_1}{\partial l_1}\Big|_o = \frac{k_1}{\mu_w}\frac{\partial p_1}{\partial l_1}\Big|_w, \tag{6}$$

$$-\frac{\partial l_1(x,t)}{\partial t} = \frac{k_1}{m_1(a_1 - a_{ocm1})\mu_o}\frac{\partial p_1}{\partial x}\Big|_o, \; l_1(x,0) = \phi_1(x), \tag{7}$$

$$p_2(x)\big|_o = p_2(x)\big|_w, \quad \frac{k_2}{\mu_o}\frac{\partial p_2}{\partial l_2}\Big|_o = \frac{k_2}{\mu_w}\frac{\partial p_2}{\partial l_2}\Big|_w, \tag{8}$$

$$-\frac{\partial l_2(x,t)}{\partial t} = \frac{k_2}{m_2(a_2 - a_{ocm2})\mu_o}\frac{\partial p_2}{\partial x}\Big|_o, \; l_2(x,0) = \phi_2(x). \tag{9}$$

Here:

P_1, P_2 - Layer pressure;

μ_o, μ_w - Dynamic viscosity coefficients;

k_1, k_2, k_Π - Layer permeability coefficients;

m_1, m_2 - Porosity coefficients;

h_1, h_2, h_Π - Layer thicknesses;

q_{1o}, q_{2o} - oil well production rate;

q_{1w}, q_{2w} - water well flow rate;

a_1, a_2 - oil saturation coefficients

l_1, l_2 - internal boundary, water-oil partition boundary.

To numerically model and solve the problem, we use the finite difference method. For this purpose, the system of differential equations (1)-(3) and the boundary conditions (4)-(9) are approximated at the k+1 time layer, resulting in the following system of finite difference equations:

$$3P_{10,j} - 4P_{11,j} + P_{12,j} = 0 \;,$$

$$a_i P_{1i-1,j} - b_i P_{1i,j} + c_i P_{1i+1,j} + d_i P_{2i,j} = -f_i, \quad i = 1,2,..,N-1,$$

$$3P_{1N,j} + 4P_{1N-1,j} - P_{1N-2,j} = 0, \qquad k+1$$

$$3P_{20,j} - 4P_{21,j} + P_{22,j} = 0,$$

$$a_i' P_{2i-1,j} - b_i' P_{2i,j} + c_i' P_{2i+1,j} + d' P_{1i,j} = -f_i', \; j = 1,2,..,M-1$$

$$3P_{2M,j} + 4P_{2M-1,j} - P_{2M-2,j} = 0.$$

Here:

$$a_i = k_{1i-0,5,j} h_{1i-0,5,j}, \quad c_i = k_{1i+0,5,j} h_{1i+0,5,j},$$

$$b_i = a_i + c_i + \frac{\Delta h^2}{\Delta \tau / 2} h_{1i,j} + \frac{\Delta h^2 k_{\Pi i,j}}{h_{\Pi i,j}} \frac{L_x L_y}{\Delta h^2}; \quad d_i = \frac{\Delta h^2 k_{\Pi i,j}}{h_{\Pi i,j}} \frac{L_x L_y}{\Delta h^2};$$

$$f_i = \frac{\Delta h^2}{\Delta \tau / 2} h_{1i,j} \hat{P}_{1i,j} + a_i \hat{P}_{1i,j-1} - (a_i + c_i)\hat{P}_{1i,j} + c_i \hat{P}_{1i,j+1};$$

$$a_i' = k_{2i-0,5,j}h_{2i-0,5,j},\quad c_i' = k_{2i+0,5,j}h_{2i+0,5,j},$$

$$b_i' = a_i' + c_i' + \frac{\Delta h^2}{\Delta\tau/2}h_{2i,j} + \frac{\Delta h^2 k_{\Pi i,j}}{h_{\Pi i,j}}\frac{L_x L_y}{\Delta h^2};\quad d_i' = \frac{\Delta h^2 k_{\Pi i,j}}{h_{\Pi i,j}}\frac{L_x L_y}{\Delta h^2};$$

$$f_i' = \frac{\Delta h^2}{\Delta\tau/2}h_{2i,j}\hat{P}_{2i,j} + a_i'\hat{P}_{2i,j-1} - \left(a_i' + c_i'\right)\hat{P}_{2i,j} + c_i'\hat{P}_{2i,j+1} - \delta_{i,j}q_{i,j};\quad i = \overline{1, N-1}.$$

$$k_{2i-0,5,j} = \frac{k_{2i-1,j} + k_{2i,j}}{2};\ k_{2i+0,5,j} = \frac{k_{2i,j} + k_{2i+1,j}}{2};\ i = 1,2,...,N-1.$$

$\hat{P}$ – The value of the pressure function at the previous time step.

The formula for the progonka method used to solve the system of finite difference equations for oil reservoirs can be written as follows: $P_{1i,j} = A_i P_{1i+1,j} + B_i P_{2i+1,j} + C_i$,

$P_{2i,j} = A_i' P_{2i+1,j} + B_i' P_{1i+1,j} + C_i' \quad i = 0,1,2,...,N-1.$ Similarly, we write the solution for water reservoirs as follows:

$$P_{1i,j} = A_i P_{1i+1,j} + B_i$$

$$P_{2i,j} = A_i' P_{2i+1,j} + B_i' \quad i = 0,1,2,...,N-1.$$

Here, the progonka coefficients are determined using the following boundary formulas[3]:

$$A_i = \frac{c_i\left(b_i' - a_i' A_{i-1}'\right)}{R_i},\quad B_i = \frac{c_i'\left(a_i B_{i-1} + d_i\right)}{R_i},$$

$$A_i' = \frac{\left(b_i - a_i A_{i-1}\right)c_i'}{R_i},\quad B_i' = \frac{c_i\left(a_i' B_{i-1}' + d_i'\right)}{R_i},$$

$$C_i = \frac{\left(a_i B_{i-1} + d_i\right)\left(a_i' C_{i-1}' + f_i'\right) + \left(a_i C_{i-1} + f_i\right)\left(b_i' - a_i' A_{i-1}'\right)}{R_i},$$

$$C_i' = \frac{\left(a_i' B_{i-1}' + d_i'\right)\left(a_i C_{i-1} + f_i\right) + \left(a_i' C_{i-1}' + f_i'\right)\left(b_i - a_i A_{i-1}\right)}{R_i},$$

$$R_i = \left(b_i - a_i A_{i-1}\right)\left(b_i' - a_i' A_{i-1}'\right) - \left(a_i B_{i-1} + d_i\right)\left(a_i' B_{i-1}' + d_i'\right)$$

$$i = 1,2,...,N-1.$$

At the boundary points, the $A_0, B_0, C_0, A_0', B_0', C_0'$ coefficient values are determined using the finite difference equations derived from the left boundary condition.

$$A_0 = \frac{\left(b_1 - 4c_1\right)}{a_1 - \left(3 - 2\Delta h\right)c_1},$$

$$B_0 = -\frac{d_1}{a_1 - \left(3 - 2\Delta h\right)c_1},$$

$$C_0 = \frac{f_1 + 2\Delta h c_1}{a_1 - \left(3 - 2\Delta h\right)c_1},$$

$$A_0' = \frac{\left(b_1' - 4c_1'\right)}{a_1' - \left(3 - 2\Delta h\right)c_1'},$$

$$B_0' = -\frac{d_1'}{a_1' - \left(3 - 2\Delta h\right)c_1'},$$

$$C_0^{'} = \frac{f_1^{'} + 2\Delta h c_1^{'}}{a_1^{'} - (3-2\Delta h) c_1^{'}}.$$

Using the above finite difference equation, along with (*i=N-1* for) and the right-hand side conditions, we will find the values of the pressure functions P_{1N} and P_{2N} at the right boundary points. By making substitutions in these equations for P_{1N} and P_{2N}, we arrive at the following equations:

$$\left[(3a_{N-1} - c_{N-1}) - (4a_{N-1} - b_{N-1}) A_{N-1} - d_{N-1} B_{N-1}^{'}\right] P_{1N} +$$

$$+\left[(4a_{N-1} - b_{N-1}) B_{N-1} - d_{N-1} A_{N-1}^{'}\right] P_{2N} = \left[d_{N-1} C_{N-1}^{'} + f_{N-1} + (4a_{N-1} - b_{N-1})\right],$$

$$\left[(3a_{N-1}^{'} - c_{N-1}^{'}) - (4a_{N-1}^{'} - b_{N-1}^{'}) A_{N-1}^{'} - d_{N-1}^{'} B_{N-1}\right] P_{2N} +$$

$$+\left[(4a_{N-1}^{'} - b_{N-1}^{'}) B_{N-1}^{'} - d_{N-1}^{'} A_{N-1}\right] P_{1N} = \left[d_{N-1}^{'} C_{N-1} + f_{N-1} - (4a_{N-1}^{'} - b_{N-1}^{'})\right].$$

By solving this system of equations for P_{1N} and P_{2N}, we obtain the following:

$$P_{1N,j} = \left(S_2 S_3^{'} - S_3 S_1^{'}\right) / \left(S_1 S_1^{'} - S_2 S_2^{'}\right),$$

$$P_{2N,j} = \left(S_3 S_2^{'} - S_1 S_3^{'}\right) / \left(S_1 S_1^{'} - S_2 S_2^{'}\right)$$

Here,

$$S_1 = \left[(3a_{N-1} - c_{N-1}) - (4a_{N-1} - b_{N-1}) A_{N-1} - d_{N-1} B_{N-1}^{'}\right],$$

$$S_2 = \left[-(4a_{N-1} - b_{N-1}) B_{N-1} - d_{N-1} A_{N-1}^{'}\right],$$

$$S_3 = \left[f_{N-1} + d_{N-1} C_{N-1}^{'} + (4a_{N-1} - b_{N-1}) C_{N-1}^{'}\right],$$

$$S_1^{'} = \left[(3a_{N-1}^{'} - c_{N-1}^{'}) - (4a_{N-1}^{'} - b_{N-1}^{'}) A_{N-1}^{'} - d_{N-1}^{'} B_{N-1}\right],$$

$$S_2^{'} = \left[-(4a_{N-1}^{'} - b_{N-1}^{'}) B_{N-1}^{'} - d_{N-1}^{'} A_{N-1}\right],$$

$$S_3^{'} = \left[f_{N-1}^{'} + d_{N-1}^{'} C_{N-1} + (4a_{N-1}^{'} - b_{N-1}^{'}) C_{N-1}\right].$$

The above finite difference formulas are also valid for the k+1 time layer.

The state of the moving boundaries is determined at each time interval by the following formulas:

$$l_1 = \hat{l}_1 - \frac{\Delta t \sigma_{1o}}{m_1 \mu_o} \frac{dP_1}{dx}; \quad l_2 = \hat{l}_2 - \frac{\Delta t \sigma_{2o}}{m_2 \mu_o} \frac{dP_2}{dx}.$$

Here $l_{i,j}$ – division limit velocity vector $l_{i,j}^{(0)} = \hat{l}_{i,j} = l(x, t_0)$.

It should be noted that the application of the propulsion method for the above-mentioned finite difference system in solving the boundary value problem ensures the absolute stability of the calculation process.

3. Results and Discussion

Based on a mathematical model and calculation algorithm, software was developed using the Matlab software tool to calculate the main indicators of gas field development in a single-layer and dynamically connected two-layer system, and computational experiments were conducted in it to take into account changes in the hydrodynamic parameters of the pressure distribution in the layer.

At high values of permeability and transmissibility coefficients, pressure distribution in the reservoir occurs rapidly. Conversely, at high values of viscosity coefficient and well flow rates, pressure distribution in the reservoir occurs more slowly. This, in turn, leads to a decrease in the reservoir's porosity coefficient.

Thus, an increase in reservoir flow rate significantly affects both pressure changes (decline) and variations in the reservoir porosity coefficient.

The calculation experiments based on the parameter values given in Figure 1 illustrate the results of gas field development over a period of 1080 days.

The first graphs at the top depict pressure distribution in the upper and lower layers of the reservoir. The first of the two middle graphs shows the pressure distribution in the cross-section of the first and second layers, while the second graph presents the dynamics of gas field development in the second layer over 360, 720, and 1080 days.

Finally, the last two graphs analyze the effect of pressure variations in the upper and lower layers on porosity, represented using contour graphs (Figure 1).

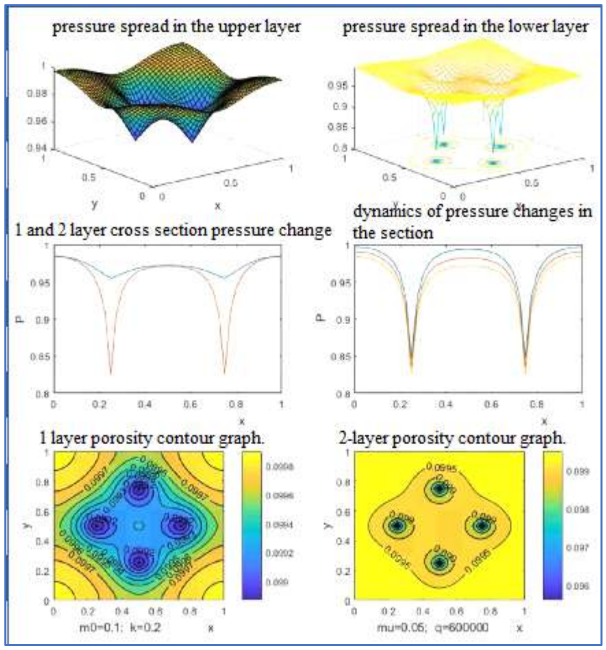

Figure 1. Graphs of pressure distribution in layers and its effect on porosity changes.

As a result of computational experiments, the influence of hydrodynamic parameters such as gas viscosity, well flow rates, permeability and porosity on the development regime of two-layer gas fields was studied. The results of computational experiments showed that these parameters are important in analyzing the main indicators of the filtration process in the development of gas fields.

4. Conclusion.

The developed software package allows for a complete study of existing and newly commissioned oil and gas wells. It is possible to conduct real-time computational experiments and obtain reliable information about the field. This developed software package is specified in the acts of its implementation.

References

[1]. Davydov L.K., Dmitrieva A.A., Konkina N.G. General hydrology. 2nd ed., revised and supplemented. – Leningrad: Gidrometizdat, 1973. – 464 p.

[2]. Abutaliev F.B. [and others]. Application of numerical methods and computers in hydrogeology. Tashkent, “Fan”, 1976.

[3]. Belman R., Kalaba R. Quasi-linearization and nonlinear boundary value problems. Mir, Moscow, 1968.

[4]. Khuzhayarov B.Kh. Macroscopic simulation of relaxation mass transport in a porous medium // Fluid Dynamics. Vol. 29, No. 5. - 2004.- P. 693-701.

[5]. Imomnazarov Sh., Mikhailov A. The Laguerre spectral method for solving dynamic problems in porous media «Bull. Nov. Comp. Center, Math. Model. in Geoph». (2020), – pp. 1–10.

[6]. Korotenko V.A., Grachev S. I., Kushakova N. P., Leontiev S.A., Zaboeva M.I., and Aleksandrov M.A., On modeling of nonstationary two-phase filtration. «IOP Conference Series Earth and Environmental Science». 2018(Russian).

[7]. Aksenov, O. A., Kozlov, M. G., Usov, E. V., Lykhin, P. A., Kayurov,N. K., & Ulyanov, V. N. Implementation of methodology to calculate three-phase equilibrium of hydrocarbons and water phase. Neft. Gas.Novacii, ((2022). 12(265)), pp. 38-43.

[8]. Nazarov.V. A. "Actual Problems of Thermal Physics and Physical Hydrodynamics" XV All-Russian School-Conference of Young Scientists with International Participation.2019. 105-113.

[9]. Akkutlu I.U., Efendiev U., Vasilyeva M.V. Multiscale model reduction for shale gas transport in fractured media. Comput. Geosci., 2016, vol. 20, no. 5, pp. 953–973.

[10]. Chetverushkin B. N., Morozov D. N., Trapeznikova M. A. et al.An Explicit Scheme for the Solution of the Filtration Problems //Mathematical Models and Computer Simulations. 2010. Vol. 2, no.6. P. 669-677.

[11]. Musakaev N. G., Borodin S. L., Belskikh D. S., Mathematical modeling of thermal impact on hydrate-saturated reservoir.«J.Comput. Methods Sci. Eng».pp 43–51. (2020).

[12]. Monteiro P.J., Rycroft Ch.H., Barenblatt G.I. A mathematical model of fluid and gas flow in nanoporous media. «Proceedings of the National Academy of Sciences of the United States of America».2012.Vol.109.№ 50.pp. 20309-20313.

[13]. Beshtokova Z.V. A difference method for solving the convection–diffusion equation with a nonclassical boundary condition in a multidimensional domain // Computer Research and Modeling -2022, -Vol. 14. -P. 559-579

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