Innovative: International Multi-disciplinary Journal of Applied Technology
(ISSN 2995-486X) VOLUME 03 ISSUE 12, 2025

Course Equations for Studying The Stability and Controllability of Wheeled Machines

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Abstract:
The controllability and stability of self-propelled energy vehicles are key parameters that determine their effectiveness in agricultural production and safety during transportation work. This work examines the course equations of wheeled machines used to analyze their stability and controllability. The influence of steering, the features of hydraulic steering systems, and their influence on the dynamic characteristics of machines are being investigated. Mathematical models are presented that take into account the rolling angles of the wheels, the transverse tilt of the machine frame, and the correction factors reflecting the influence of external factors on the stability of the movement. The obtained results can be used to optimize the structural parameters of steering and improve the safety of wheeled vehicles during their operation on public roads.

Keywords: Course movement, controllability, stability, steering, drift angles, transverse tilt, wheeled machines.

1. Introduction

One of the important operational properties of self-propelled energy vehicles is controllability and stability of movement. The productivity and agrotechnical indicators of the machine in agricultural production, as well as the safety of movement when using energy vehicles with trailers in transport operations, largely depend on these properties [1], [2], [3].

Under other equal conditions, high accuracy and ease of machine control during various technological operations in agricultural production and safety of movement in transport operations can only be ensured with good control and high stability of the self-propelled energy vehicle's movement.

One of the important factors influencing the controllability and stability of self-propelled energy vehicles is steering. Currently, the predominant type of steering used in tractors, road construction, and agricultural machinery is volumetric hydrostatic steering.

The absence of mechanical transmissions ensures freedom of arrangement and reduces the machine's metal consumption. At the same time, this type of steering control is characterized by the "slip" of the steering wheel and greater flexibility compared to mechanical steering controls with amplifiers.

Within the framework of G.G. Rasulov conducted research on the controllability and stability of MTAs during sowing and inter-row cultivation of cotton, demonstrating fundamental differences in the nature of the controlled wheel's perception of disturbances under different types of steering, and developed and implemented recommendations for improving the controllability and stability of 3-wheeled cotton-modified tractors [4], [5], [6].

Materials and Methods

This study employed a theoretical and analytical modeling approach to investigate the controllability and stability of self-propelled wheeled energy vehicles. The analysis was carried out by formulating the course motion equations of a wheeled machine based on force and moment projections along the coordinate axes connected to the machine's center of mass. The model assumes constant vehicle speed and small steering and drift angles, allowing the sine and tangent values of these angles to be approximated by their arguments. The mathematical model was further supplemented with kinematic relations to determine the displacement angles of the front and rear wheels, including the effects of wheel inclination, transverse frame tilt, steering geometry, and rear-wheel drive conditions. In addition, correction coefficients were introduced to account for variations in vertical wheel load, tractive force on the driving wheels, and soil deformation properties in the lateral direction. The steering system was represented through the characteristics of the hydrostatic steering drive, while suspension kinematics and transverse tilt equations were used to describe the dynamic behavior of the wheeled machine under different operating conditions [7], [8].

Results and Discussion

B. Turgunbayev conducted research on the selection of a support-turning device for a self-propelled power vehicle with a rear-placed steered wheel.

Transport tractors are also equipped with hydraulic-volume steering, which, when transporting raw cotton from the fields to the procurement points, pull 3-4 trailers. The working conditions of a transport tractor differ significantly from the working conditions of a tractor with mounted units in the inter-row spaces. This is explained not only by the higher range of working speeds of the transport tractor, but also by the requirements for it arising from the conditions of their use on public roads [9], [10]. From this it follows that the transport tractor must meet certain requirements for maneuverability and stability of movement, which largely depend on the characteristics of the steering control. The "slip" of the steering wheel, the inaccurate correspondence of the steering wheel's rotation angles to the steering wheel's rotations, and some other features of this type of steering can lead to insufficient safety of the tractor train when moving on public roads under conditions of dense traffic flow.

The article is devoted to theoretical research on developing mathematical models of self-propelled wheeled power vehicles, compiling a mathematical description of steering using the experimentally obtained transmission function of the steering hydraulic drive, and calculating the parameters of the rotating hydraulic cylinders of the driven wheels [11], [12], [13].

The third chapter of the dissertation describes the testing methodology, measuring and recording equipment.

The fourth chapter of the dissertation is devoted to the analysis of the results of experimental studies, the generalization of the results of theoretical and experimental studies, as well as the development of conclusions and recommendations.

The equations of the course motion of a wheeled machine in projections onto the coordinate axes, connected by the center of mass of the machine, have the form: Figure 1.

$$m(\dot{v}_y + v_x\omega_z) = \Sigma Yi$$
$$J_z\omega_z = \Sigma M_{zi}, \qquad (1)$$

where m-mass of the machine;

V_x, $\dot{V}_y$ – projections of the translational velocity and transverse acceleration of the machine's center of mass on the X and Y axes;

ΣY_i – projections of the principal vector of external forces onto the axis Y.

ω_z – projection of the machine's angular velocity onto the axis Z.

ΣM_{zi} – principal moment of external forces relative to the axis Z.

J_z – moment of inertia of the machine relative to the vertical axis.

These equations are written assuming the machine's speed is constant, and the rotation and departure angles of the wheels are small, i.e., they do not exceed 10. This allows us to take the sines and tangents of the angles as equal to their arguments. It should be considered that such assumptions are valid for solving problems of stability and controllability of wheeled machines. To solve the problem, the system of equations (1) must be supplemented with equations for the displacement angles of the wheels, which can be obtained from kinematic relations.

The average yield angle of the front wheel tires is determined by the equation:

$$\delta_i = \frac{1}{v_x}(v_y + a\omega) - \theta_{cp}(2)$$

where θ_{cp} – the average angle of rotation of the driven wheels;;

v_y- transverse velocity of the center of mass of the machine;

a- the distance from the center of mass of the machine to the front axis.

Average rotation angle of the driven wheels

$\theta_{cp} = (\theta_Л + \theta_П)/2$

Consideration of the additional lateral force arising from the wheel's inclination caused by the bend of the bridge or body is most conveniently represented in the form of additional tire displacement, which can be expressed as follows:

$$\delta_\phi = \phi_i \cdot \lambda_i \qquad (3)$$

where φ_i – angle of inclination of the wheels caused by the slope of the bridge or frame of the machine; λ_i - the equivalence coefficient between the wheel's slope and the corresponding drift angle.

Then the average displacement angle of the front wheel tires will be determined from the expression

$$\delta_1 = \frac{1}{v_x}(v_y + a\omega) - \theta_{cp} + \lambda_1\phi_i\phi \qquad (4)$$

Average drive angle of the rear wheel tires, determined taking into account the wheel inclination:

$$\delta_2 = \frac{1}{V_x}(Vy + в\omega) + \lambda_2\phi_0, \qquad (5)$$

where φ_0 – crest angle of the machine frame; λ_2 has the same meaning as the coefficient λ_1.

The drive angles of the front and rear wheels for a machine with rear-wheel drive are determined by the formulas shown in Figure 2

$$\delta_1 = \frac{V_y - a\cdot\omega}{V_x} \tag{6}$$

$$\delta_2 = \frac{V_y + b\cdot\omega}{V_x} \tag{7}$$

When solving the systems of equations (6) and (7), it is advisable to consider the influence of the change in the vertical load on the wheels and the tractive force on the driving wheels on the drag coefficients. Additionally, for cases of modeling machine movement along deformable soil, consider the influence of soil deformation properties in the lateral direction [14], [15].

Based on the results of their work, the equivalent resistance coefficient of the tire can be determined as follows:

$$K = q_N \cdot q_T \cdot q_\Gamma \tag{8}$$

where: q_N – Takes into account the influence of vertical load on the coefficient of resistance to leakage;

q_T – takes into account the influence of the tractive force;

q_Γ – takes into account the deformation properties of the soil in the lateral direction.

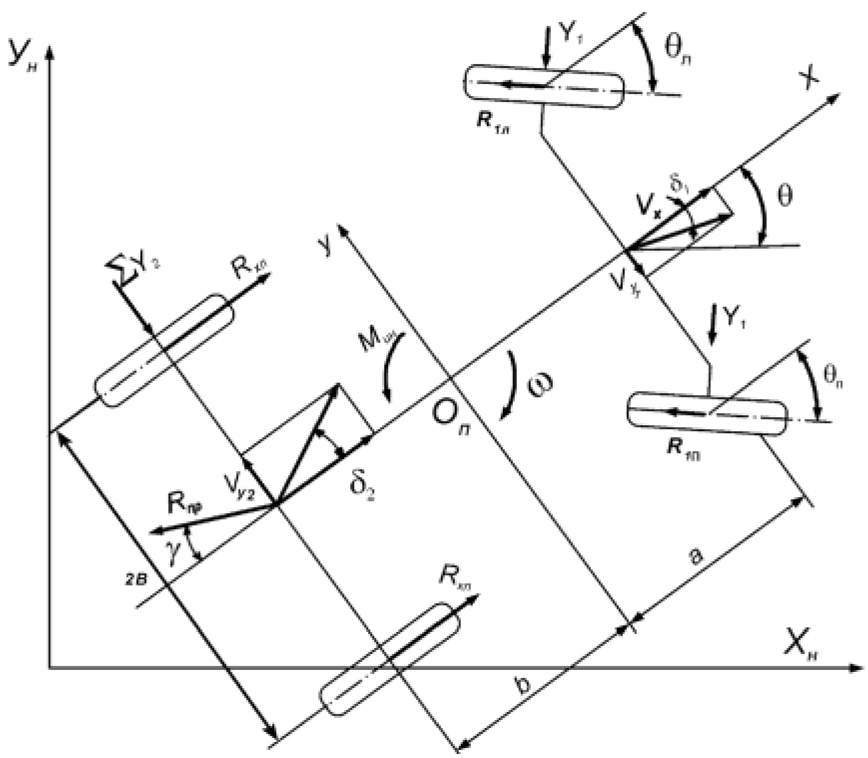

Figure 1. Calculation scheme of a wheeled machine with front controller and wheels.

These coefficients are determined by the following formulas.

$$q_N = 1 - 0{,}6\cdot(\Delta R_Z/R_Z) + 0{,}4\cdot(\Delta R_Z/R_Z) \tag{9}$$

where $\Delta R_Z = R_z - R_{zo}$;

(R_z – current value of vertical load;

R_{zo}- vertical load corresponding to the maximum value of the drag coefficient.)

The correction factor, which takes into account the influence of the tractive force, is determined by the formula,

$$q_T = \sqrt{(1 - R_x/\phi \cdot R_Z)^2} \qquad (10)$$

where R_x - tangential reaction on the driving wheel

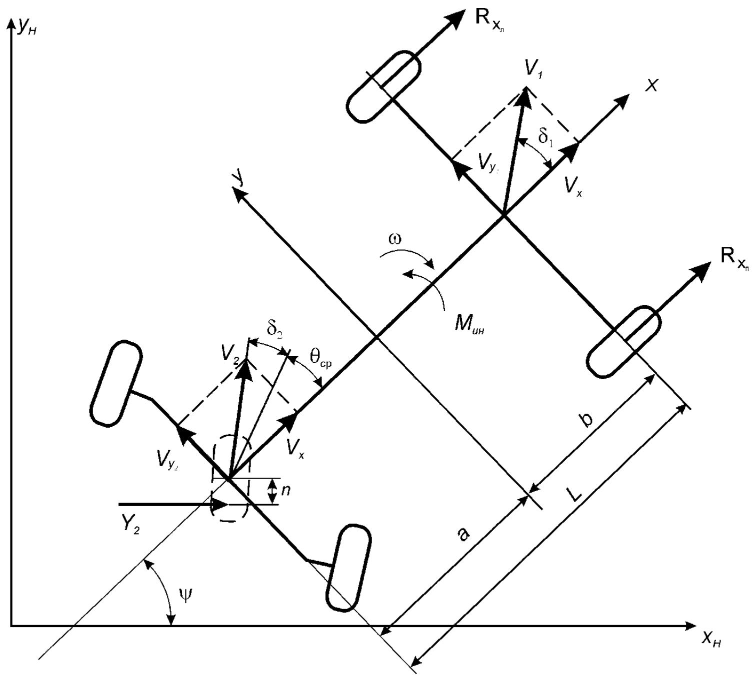

Fig.2. Calculation scheme for determining the departure angles of a machine with given control wheels.

The correction coefficient $q_Г$ is determined by the formula

$$q_Г = 1 + 2К_\delta/С_Г \cdot b \cdot l^2 \qquad (11)$$

where b, l - tire imprint width and length;

$C_Г$ – soil stiffness in the lateral direction.

The lateral stiffness of the tire during rolling on deformable soil is determined by the formula

$$С_Э = \big((С_Ш \cdot С_Г)/С_Ш + С_Г\big) \qquad (12)$$

where $C_Э$ – side stiffness of the tire.

Expressions for the angles of inclination of the wheels * relative to the machine frame can be obtained based on the suspension kinematics. The front axle beam of 4-wheeled tractors has a longitudinal hinge connecting the front axle to the tractor frame [14]. In this case, the angles of inclination of the front wheels will be equal to the angle of the transverse tilt of the bridge connected to the bridge by a candle suspension.

With such a suspension of the front axle, the angles of inclination of the wheels relative to the machine frame are determined by the formula:

$$\phi_К = \lambda_\phi(\phi_0 - \phi_М) \qquad (13)$$

where φ_0 – Crest angle

In addition, the transverse tilt of the base can lead to the rotation of the bridge beam in the horizontal plane (kinematic wheel drift).:

$\theta_K = \lambda_\theta(\phi_0 - \phi)$, (14)

where $\boldsymbol{\lambda_\varphi}$, $\boldsymbol{\lambda_\theta}$ - are the proportionality coefficients between the slope angles and the rotation of the bridge beam axis in plan during machine frame slope.

For a wheeled machine connected to the front spring-loaded bridge, which is hinged to the machine frame, (4-wheeled tractor diagram) the following equations can be written for the transverse tilt angles, Fig. 3.

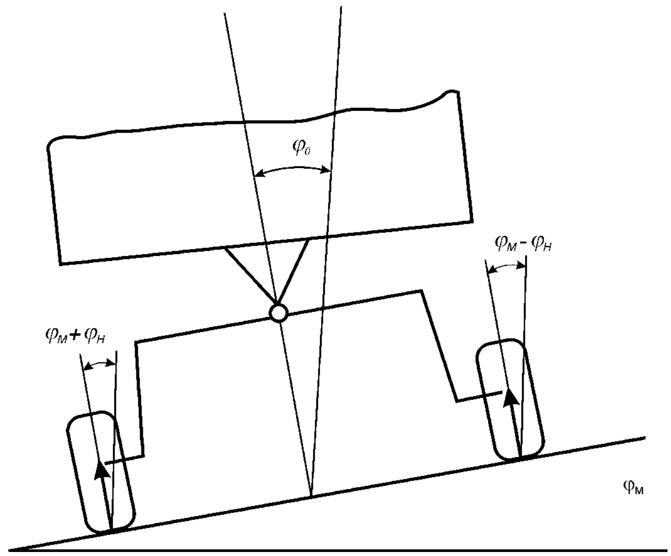

Fig.3 calculation scheme of the machine's tilted crest

$(J_x - J_{m_1})\ddot{\phi} + K_{\phi_1}(\dot{\phi} - \dot{\phi}_{m_1}) + K_{\phi_2}(\dot{\phi} - \phi_{m_1}) + C_{\phi_1}(\phi - \phi_{m_1}) - mgh_c\phi + h_c(Y_1 + Y_2) + 2C_{\text{ш}} d_k^2 \phi_{q^2} = 0$ 15

$$J_{m_1}\ddot{\phi}_{m_1} - K_{\phi_1}(\dot{\phi} - \phi_{m_1}) - C_1(\phi - \phi_{m_1}) + 2C_{\text{ш}} d_k^2(\phi_{m_1} - \phi_{q_1}) = 0,$$

where **Cш** – radial stiffness of tires;

dk - tracks of the machine wheels;

$\boldsymbol{\varphi_{q1}}$ and $\boldsymbol{\varphi_{q2}}$ and - average angles of the road's transverse slope;

$\boldsymbol{J_x}$ – moment of inertia of the machine relative to the X axis;

$\boldsymbol{J_m}$ – moment of inertia of the controlled bridge relative to the horizontal axis of the bridge suspension to the tractor frame

$\boldsymbol{K_\varphi}$ - crest resistance coefficient, proportional to the crest's angular velocity;

$\boldsymbol{h_C}$ - height of the tractor's center of mass.

For the case of motion along a smooth horizontal road, the equations for the angles of transverse tilt are significantly simplified, since it is possible to neglect the oscillations of the bridge.

For this case, the machine frame crest equation has the form:

$$\dot{J}_x\ddot{\phi} + k\dot{\phi} + C_\phi\phi + h_c(Y_1 + Y_2) = 0, \quad (16)$$

where $\dot{J}_x = J_x - J_{m_1}$;

$$\boldsymbol{C_\phi = \frac{C\phi_1 C_{\text{ш}1}}{C\phi_1 + 2C_{\text{ш}} d_{\text{к}}^2} - mgh_C}. \quad (17)$$

φ_H- structural deformation of the wheel.

Conclusion

The developed course motion equations provide a systematic basis for analyzing the stability and controllability of wheeled machines, particularly self-propelled energy vehicles equipped with hydrostatic steering systems. The study shows that the directional stability of a wheeled machine is influenced not only by steering characteristics, but also by wheel drift angles, transverse frame inclination, vertical load distribution, tractive force, tire lateral stiffness, and soil deformation properties. By incorporating these factors into the mathematical model, it becomes possible to more accurately evaluate the dynamic response of the machine during agricultural and transport operations. The proposed approach can be used to optimize steering parameters, improve the design of hydraulic steering mechanisms, and enhance the operational safety of wheeled vehicles, especially when they are used with trailers on public roads. Future studies should include broader experimental validation under various speeds, road surfaces, soil conditions, and loading configurations to strengthen the practical applicability of the model.

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