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ESTIMATING FUZZY PARAMETERS OF SPATIAL REGRESSION BY ST-DECOMPOSITION METHOD

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Abstract:
This study evaluates the Fuzzy Spatially Lagged Exogenous (FSLX) model for analyzing ambiguous spatial data, featuring a symmetric triangular fuzzy response variable, deterministic crisp explanatory variables, and symmetric triangular fuzzy parameters, estimated via the Symmetric Triangular ST-decomposition matrix method. The model was empirically applied to 2018 traffic fatality data and its contributing factors across six Iraqi governorates. Results demonstrate the efficacy of the ST-decomposition approach in disassembling the fuzzy system into core and spread components to simplify estimation. A statistically significant positive relationship was identified between fatalities and rollover/pedestrian accidents, with Moran's I test confirming significant spatial autocorrelation. Furthermore, spatial lag parameters revealed varying geographical spillover effects among neighboring governorates, maintaining parameter stability and effectively encapsulating observational uncertainty within the fuzzy bounds. Future extensions to spatio-temporal and advanced fuzzy spatial models are recommended.

Keywords: Fuzzy Spatial Regression; FSLX Model; ST-Decomposition Method; Symmetric Triangular Fuzzy Numbers; Spatial Autocorrelation

Introduction

Statistical regression analysis serves as a fundamental analytical tool for explaining functional relationships between dependent and explanatory variables. In handling complex empirical data, two major challenges arise uncertainty which is addressed by Fuzzy Linear Regression, and spatial dependency," which violates the assumption of independent observations inherent in Ordinary Least Squares (OLS). Consequently, the Fuzzy Spatial Regression model emerges as an advanced framework that simultaneously handles ambiguous data and models geographic spatial interactions. The (SLX) model holds particular importance for measuring spatial spillover effects via the spatial weight matrix (W). Integrating fuzzy logic into the (SLX) framework using the Symmetric Triangular ST-decomposition method enables disassembling the fuzzy system into its core and spread components. This facilitates precise parameter estimation and enhances forecasting efficiency within an uncertain environment [1]. To demonstrate this analytical capability, the model was applied to 2018 empirical data on traffic accident fatalities and contributing factors across six Iraqi governorates (comprising 38 spatial observations). This empirical application evaluates spatial lag effects while formulating variables and parameters as Symmetric Triangular Fuzzy Numbers to effectively capture observational uncertainty [2].

Research Objective:

This study presents a theoretical and empirical framework for the Symmetric Triangular (ST_decomposition}) matrix method in estimating spatially lagged linear regression parameters under triangular fuzzy variables and parameters. The methodology is applied to 2018 traffic fatality data across six Iraqi governorates to demonstrate the model's explanatory power [3].

Fuzzy Set Theory:

Fuzzy Set Theory is considered one of the contemporary mathematical and statistical foundations developed to address uncertainty and ambiguity inherent in data. In doing so, it overcomes the limitations of classical binary logic, which restricts outcomes to only two polar values. In traditional studies, statistical models often lack sufficient flexibility to represent gradual real-world phenomena, particularly in spatial analysis. Since the concept was first introduced by introduced by "Lotfi Zadeh" in 1965, it has provided a mathematical framework that allows an element to belong to a set with varying degrees of membership, falling within the closed interval [4]. The need for this theory arose to transcend the constraints of rigid, ready-made templates when interpreting economic, social, and environmental variables subject to relative estimation. With the expansion of statistical research, reliance on fuzzy logic in advanced modeling has increased; the "membership function" grants researchers the capacity to characterize the partial membership of elements using precise real values. This approach constitutes a fundamental core in formulating fuzzy regression models, as it contributes to enhancing parameter estimation accuracy compared to traditional methods. Consequently, this has rendered it an indispensable tool for constructing spatial fuzzy regression models, given its high capacity to address spatial heterogeneity and improve estimation quality when comparing different methodologies in ambiguous environments [5].

Membership Function:

The membership function is the cornerstone of fuzzy set theory, serving as a statistical tool that maps qualitative or ambiguous data into quantitative values within the interval [6], thereby bypassing rigid binary logic. Mathematically, it expresses fuzzy sets as ordered pairs to model "partial membership" in overlapping phenomena, capturing data variance rather than assuming absolute homogeneity. Selecting the function’s shape is a core statistical decision based on the variable's nature and ambiguity. Ultimately, its inferential significance lies in structuring uncertainty within analytical models, enhancing the efficiency and robustness of fuzzy parameters when representing highly dispersed or low-precision data.[7]

Types of Fuzzy Sets:

Fuzzy sets are classified into two main types:[8]

1. Discrete Fuzzy Sets: The universe of discourse consists of a finite or countable number of distinct, isolated values.

$$\tilde{A} = \left\{ \sum_{i=1}^{n} \left(\mu_{\tilde{A}}(x_i) / x_i \right) \right\} \qquad \dots\dots\dots\dots (1)$$

2. Continuous Fuzzy Sets : The universe of discourse is a continuous range or interval

$$\tilde{A} = \left\{ \int \mu_{\tilde{A}}(x) / x \right\} \qquad \dots\dots\dots\dots (2)$$

Materials and Methods

Types of Membership Functions:

Membership functions in fuzzy set theory vary according to their different shapes and mathematical properties, which are determined by the nature of the data and the purpose of the statistical analysis. This diversity contributes to increasing the flexibility and accuracy of fuzzy modeling. Examples include the Triangular Membership Function, the Trapezoidal Membership Function, and the Gaussian Membership Function.

Triangular Membership Function:

This is considered one of the most widely used functions in fuzzy logic due to its simplicity. It is utilized when precise data is absent or when satisfying an approximate behavior for the variable is sufficient. A triangular fuzzy number is represented by a triplet of ordered values:

$\tilde{A} = (l, c, u) \quad , \quad l < c < u$

Where the value l represents the lower bound (minimum value), the value c represents the most typical value (the peak of the triangle), while the value u represents the upper bound (maximum value) .

The membership function is mathematically formulated as follows:

$$\mu_A(x) = \begin{cases} \frac{x-l}{c-l}, & l \leq x \leq c \\ 1 \quad , & x = c \\ \frac{u-x}{u-c}, & c \leq x \leq u \\ 0 \quad , & o.w \end{cases} \qquad \text{...........(3)}$$

The shape of the membership function for a triangular fuzzy number is illustrated below:

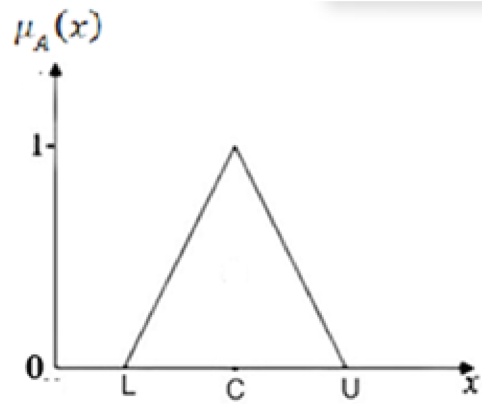

Figure 1. Triangular Fuzzy Number

Spatial Linear Regression:

Spatial linear regression provides an advanced statistical framework that overcomes the limitations of traditional models which assume the independence of observations. It acknowledges that geographic data is inherently interconnected, as values in adjacent locations tend to exhibit similarity-a phenomenon known as "spatial dependence" (or spatial autocorrelation). The significance of this model lies in its ability to account for "omitted variables" (such as environmental factors or shared cultural traits) and quantify them into numerical data that reflects the actual reality of the phenomena under study.

General Spatial Model (GSM):

The General Spatial Model (GSM) is considered one of the comprehensive models in spatial econometrics, as it simultaneously integrates two main types of spatial effects: spatial dependence in the dependent variable (via the autoregressive parameter λ) and spatial dependence in the error term (via the parameter ρ). This integration offers superior flexibility in explaining interconnected relationships across geographical locations and modeling the complex spatial correlations that may arise in empirical data. The functional form of the GSM is specified as follows:

$$Y = \lambda WY + X\beta_1 + WX\beta_2 + u \quad , \quad |\lambda| < 1 \qquad \ldots\ldots\ldots\ldots(5)$$

$$u = \rho Wu + e \quad , \quad |\rho| < 1$$

Where Y is an $(n \times 1)$vector of the dependent variable. X is an $(n \times \ k)$ matrix of non-stochastic regressors (independent variables).W is an $(n \times n)$ spatial weight matrix.$\beta_1\beta_2$are $(k \times 1)$ vectors of parameters to be estimated. λ, ρ are the spatial regression parameters. u is the spatially correlated error term. e is a $(n \times 1)$ vector of random errors, given that $e/X = i.i.d.N(0, \sigma_{en}^2 I_n)$. Alternatively, the model can be reformulated as:

$Y = \lambda WY + Z\beta + u \quad , \quad |\lambda| < 1$

$u = \rho Wu + e \quad , \quad |\rho| < 1$

Where $Z = [X \quad WX]$ and $\beta = [\beta_1 \quad \beta_2]$. This specific specification is referred to as the Spatial Autoregressive Model.

While there are numerous spatial econometric models that can be formulated, in this research, we employ the Spatially Lagged X's Model (SLX), which arises under the restriction $(\rho = \lambda = 0)$. The structural form of this model is given by:

$$Y = X\beta_1 + WX\beta_2 + e \qquad \dots\dots\dots\dots (6)$$

General Fuzzy Spatial Model (GSM):

As an extension of the general spatial model, the General Fuzzy Spatial Model (GFSM) is introduced to handle real-world data characterized by vagueness or uncertainty. In many cases, economic or environmental variables cannot be expressed using crisp, precise numbers. Therefore, Fuzzy Logic is employed in this model to transform variables and parameters into fuzzy formulations, enabling spatial modeling with higher accuracy and better simulation of uncertain real-world data. The model structure is formulated as follows [6]:

$$\tilde{Y} = \lambda W\tilde{Y} + X\tilde{\beta}_1 + WX\tilde{\beta}_2 + \tilde{u} \quad , \quad |\lambda| < 1 \qquad \dots\dots\dots\dots (7)$$

$$\tilde{Y} = \lambda W\tilde{Y} + Z\tilde{\beta} + \tilde{u} \quad , \quad |\lambda| < 1$$

$\tilde{u} = \rho W\tilde{u} + \tilde{e} \quad , \quad |\rho| < 1$

Where $Z = [X \;\; WX]$and $\tilde{\beta} = [\tilde{\beta}_1 \;\; \tilde{\beta}_2]$.

In the Fuzzy Spatially Lagged X's (FSLX) Model, where [$\rho = \lambda = 0$].

$$\tilde{Y} = X\tilde{\beta}_1 + WX\tilde{\beta}_2 + \tilde{e} \qquad \dots\dots\dots\dots (8)$$

Where $\tilde{Y} = (Y_l, Y_c, Y_u)$ و$\tilde{\beta}_1 = (\beta_{1l}, \beta_{1c}, \beta_{1u})$, $\tilde{\beta}_2 = (\beta_{2l}, \beta_{2c}, \beta_{2u})$.

Results

Symmetric Triangular ST-decomposition Method:

The Symmetric Triangular ST-decomposition method for estimating the parameters of a fuzzy linear regression model is based on representing the fuzzy parameters in the form of symmetric triangular fuzzy numbers (STFNs). Under this framework, the fuzzy spread around the central value is assumed to be symmetric on both sides. This representation aims to simplify the mathematical structure of the fuzzy model while preserving the essential statistical properties of the parameters. The method operates on the principle of decomposing the symmetric triangular fuzzy number into two core components: The central value component (core) [9]. The uncertainty-related component (spread). This decomposition allows for the isolation of the deterministic effect from the fuzzy effect within the model. Such a disassembly provides an effective approach to characterizing the relationship between the dependent variable and the independent variables within a symmetric fuzzy framework, thereby avoiding additional complexity in parameter representation. The ST-decomposition method is characterized by its simplicity and clarity in statistical formulation, making it highly suitable for applications where uncertainty is assumed to be symmetric around the central values of the parameters. Furthermore, its reliance on symmetric triangular fuzzy numbers renders it one of the most appropriate methods for analyzing fuzzy models that require estimation stability and straightforward interpretation of statistical results. The model formulation can be rewritten in matrix form as:[10] [11]

$\tilde{Y} = Z\tilde{\beta} + \tilde{e} \quad , \quad Z = [X \quad WX] \;\; , \; \tilde{\beta} = [\tilde{\beta}_1 \quad \tilde{\beta}_2]$

This transformation represents a fundamental step that facilitates handling the fuzzy spatial model using estimation tools specific to fuzzy linear regression. By applying the Ordinary Least

Squares (OLS) method, the estimation equations for the fuzzy spatial model are obtained in the following form:

$$Z'Z\tilde{\beta} = Z'\tilde{Y} \qquad \text{……….(9)}$$

This formulation represents the spatial expansion of the classical estimation equation. Consequently, the fuzzy spatial linear regression analysis is mapped into a fuzzy linear system that can be solved using the ST-decomposition technique, where:

$$Z'Z = ST \qquad \text{……….(10)}$$

Where S is a symmetric matrix and T is an upper triangular matrix. The resulting system from the previous equation is solved by finding the inverse of both matrices S and T based on the steps of the ST decomposition [12].

Let us assume we have a fuzzy number $\tilde{u}$defined via its α -level cuts such that:

$$\tilde{u} = \left(\underline{u}(\alpha), \overline{u}(\alpha)\right)$$

Where $0 \leq \alpha \leq 1$ For this representation to be valid, the following conditions must be satisfied:

First: The function $\underline{u}(\alpha)$ must be continuous and monotonically increasing with respect to α.

Second: The function $\overline{u}(\alpha)$must be continuous and monotonically decreasing with respect to α.

Third: The condition $\underline{u}(\alpha) \leq \overline{u}(\alpha)$must hold for all $\forall\ \alpha \in [0,1]$.

In the case of adopting triangular fuzzy numbers to represent the parameters of the spatial model, the membership function of the triangular fuzzy number $\tilde{u} = (u_l, u_c, u_u)$ is given by:

$$\mu_{\tilde{u}}(x) = \begin{cases} \dfrac{x - u_l}{u_c - u_l}, & u_l \leq x \leq u_c \\ \dfrac{u_u - x}{u_u - u_c}, & u_c \leq x \leq u_u \\ 0\ , & o.w \end{cases} \qquad \dots\dots\dots\dots\dots\dots(11)$$

Based on the interval/cut representation, the triangular fuzzy number can be rewritten $\tilde{u} = \left(\underline{u}(\alpha), \overline{u}(\alpha)\right)$

Where $\overline{u}(\alpha) = u_u - (u_u - u_c)\alpha$ and $\underline{u}(\alpha) = (u_c - u_l)\alpha + u_l$

Into an equivalent fuzzy linear system using the lower and upper bounds representation of the parameters, which aligns with the Symmetric Triangular Decomposition (ST-Decomposition) technique to take the form:

$$AB = Y \qquad \dots\dots\dots\dots\dots.(12)$$

Where $B = \begin{bmatrix} \underline{\beta} \\ -\overline{\beta} \end{bmatrix}$, $Y = \begin{bmatrix} \underline{Y} \\ -\overline{Y} \end{bmatrix}$

The matrix A can be written in the block form:

$$A = \begin{bmatrix} M & N \\ N & M \end{bmatrix}$$

Such that $M, N \geq 0$، where the matrices M and N represent the positive and negative parts of the explanatory matrices resulting from the independent variables and the spatial lag variables WX in the (FSLX) model [13]. Consequently, the fuzzy linear system can be rewritten as:

$$\begin{bmatrix} M & N \\ N & M \end{bmatrix} \begin{bmatrix} \underline{\beta} \\ -\overline{\beta} \end{bmatrix} = \begin{bmatrix} \underline{Y} \\ -\overline{Y} \end{bmatrix}$$

One of the properties of matrix A is that it is non-singular if its determinant is non-zero, that is, if:

$$B = M - N \ \ , \ \ B = M + N$$

are non-singular. In this case, the inverse of matrix A can be found, which takes the form:

$$A^{-1} = \begin{bmatrix} P & Q \\ Q & P \end{bmatrix}$$

Using the ST-Decomposition technique, matrix A is decomposed into the product of two symmetric matrices:

$Y = S\,T\,A$ …………(13)

Which allows finding the fuzzy solution for the system through the inverse of the resulting matrices. After reformulating the fuzzy response vector into the form:

$$Y = \begin{bmatrix} Z'\tilde{Y} \\ -Z'\tilde{Y} \end{bmatrix}$$

The parameter vector of the fuzzy spatial linear regression model of type (FSLX) can be estimated using the relationship:

$$\hat{B}_{ST} = T^{-1}S^{-1}Y = T^{-1}S^{-1}\begin{bmatrix} Z'\tilde{Y} \\ -Z'\tilde{Y} \end{bmatrix} \quad \text{…………..(14)}$$

Where $\hat{B}_{ST}$ represents the fuzzy estimates for the parameters of the FSLX model, which account for the indirect spatial effects represented by the matrix W, in addition to addressing the fuzzy uncertainty associated with the parameters [14].

Moran's I Index:

Measures spatial autocorrelation to determine if geographic data is clustered, dispersed, or random, helping build spatial models.

$$\mathrm{I} = \frac{\mathrm{n}(\mathrm{e}'\mathrm{We})}{\mathrm{S}_0(\mathrm{e}'\mathrm{e})} \quad \text{… … … … (15)}$$

Where n is sample size, e is an $(n \times 1)$ vector of random errors, and ($\mathrm{S}_0 = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{w}_{ij}$) is the sum of elements in the spatial weights matrix (W).

Hypotheses(Z-test)

$\mathrm{H}_0 = \lambda = 0 \quad , \quad \rho = 0$ $\qquad$ $(\textit{No spatial dependence})$

H_1 : at lest one of $\lambda \neq 0$ or $\rho \neq 0$ $\qquad$ $(\textit{Spatial dependence exists})$

Test Statistic Z: $\mathrm{Z} = \frac{\mathrm{I}-\mathrm{E(I)}}{\sqrt{\mathrm{V(I)}}}$ … … … … (16)

Mean: $\mathrm{E(I)} = \frac{\mathrm{n}(\mathrm{tr}(\mathrm{MW})}{\mathrm{S}_0(\mathrm{n}-\mathrm{k})}$ … … … … (17)

Variance:

$$\mathrm{V(I)} = \frac{\mathrm{tr}(\mathrm{MWMW}') + (\mathrm{tr}(\mathrm{MW}))^2 + (\mathrm{tr}(\mathrm{MW})^2)}{(\mathrm{n}-\mathrm{k})(\mathrm{n}-\mathrm{k}+2)}\left(\frac{\mathrm{n}}{\mathrm{S}_0}\right)^2 - \left(\mathrm{E(I)}\right)^2 \quad \text{… … (18)}$$

Where ($\mathrm{M} = \mathrm{I} - \mathrm{X}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}'$) is a symmetric square matrix, $tr(M)$ is the trace (sum of main diagonal elements) and k is the number of independent variables.

Compare calculated Z with tabular $Z_{(\alpha\,/\,2)}$. If significant, spatial dependence exists (requires spatial regression models); otherwise, use the General Linear Model (GLM).[8]

Spatial Weights Matrix (Calculate Spatial Weights Matrix):

The core difference between traditional and spatial statistics is the latter's ability to handle geographic dimensions (points, lines, or polygons) and analyze environmental links based on the "neighborhood principle." This relationship is statistically controlled via the Spatial Weights Matrix (W), which is a square matrix whose dimensions depend on spatial adjacencies between locations (i) and (j), where its diagonal elements equal zero. This matrix includes several types, such as Linear Contiguity, where the element value $W_{ij} = 1$ when entities i and j are adjacent and share a common linear feature within the area of interest, and $W_{ij}= 0$ otherwise. And is the Row-Standardized Weights Matrix $\mathrm{W}^{\mathrm{std}}$, also called the standardized matrix, where the sum of each row equals one, computed based on the binary contiguity matrix using the formulation:[15]

$$\mathrm{W}^{\mathrm{std}} = \left(\mathrm{w}_{ij}\right) = \begin{bmatrix} \frac{\mathrm{w}_{ij}}{\sum_i \mathrm{w}_{ij}} & \text{if i neighbor j} \quad 0 < \mathrm{W}^{\mathrm{std}} \leq 1 \\ 0 & \text{otherwise} \end{bmatrix} \quad \text{… … … (19)}$$

Application:

Real spatial data was utilized for the empirical part, representing the number of traffic accident fatalities for the year 2018 across six Iraqi governorates: (Baghdad, Nineveh, Al-Anbar, Saladin, Kirkuk, and Diyala). The study encompassed 38 observations derived from previous research.

Spatial locations were projected onto the map of Iraq using Geographic Information Systems (GIS) to delineate the administrative boundaries for each site. These data represent the number of fatalities resulting from various traffic accidents (such as collisions, rollovers, and run-overs). Accordingly, the dependent and independent variables for the study were defined, where Y represents the dependent variable (the number of traffic accident fatalities), while the independent variables represent the number of collision accidents X_1, the number of rollover accidents X_2,, and the number of run-over accidents X_3. Since this study addresses the approach of Symmetric Triangular ST-decomposition in estimating spatial regression parameters where the independent variables have crisp values and the dependent variables are fuzzy, the dependent variable Y was fuzzified into triangular fuzzy numbers using MATLAB software, as illustrated in Table (1) below:

Table 1. Traffic accident data for six governorates with crisp independent variables and a fuzzified dependent variable

Locations	Y=Fuzzy Number	X1	X2	X3
1	A6=[40,48,56]	10	0	65
2	A3=[16,24,32]	16	4	13
3	A2=[8,16,24]	13	2	10
4	A1=[0,8,16]	2	1	4
5	A1=[0,8,16]	10	3	0
6	A1=[0,8,16]	3	1	7
7	A2=[8,16,24]	13	2	1
8	A5=[32,40,48]	46	12	51
9	A4=[24,32,40]	25	20	22
10	A3=[16,24,32]	15	13	8
11	A2=[8,16,24]	57	1	50
12	A1=[0,8,16]	5	3	2
13	A2=[8,16,24]	10	5	4
14	A1=[0,8,16]	6	1	0
15	A1=[0,8,16]	1	2	1
16	A1=[0,8,16]	2	3	1
17	A10=[72,80,88]	196	13	134
18	A3=[16,24,32]	47	9	1
19	A9=[64,72,80]	60	15	13
20	A1=[0,8,16]	50	12	10
21	A2=[8,16,24]	10	9	3
22	A8=[56,64,72]	100	15	125
23	A1=[0,8,16]	40	12	50
24	A2=[8,16,24]	35	8	30
25	A1=[0,8,16]	25	5	45
26	A4=[24,32,40]	120	18	110
27	A1=[0,8,16]	56	10	72
28	A5=[32,40,48]	22	5	18
29	A2=[8,16,24]	7	1	9
30	A1=[0,8,16]	5	0	4
31	A6=[40,48,56]	15	2	14
32	A2=[8,16,24]	156	6	24
33	A1=[0,8,16]	5	1	0
34	A1=[0,8,16]	20	9	5
35	A1=[0,8,16]	7	0	1

Locations	Y=Fuzzy Number	X1	X2	X3
36	A1=[0,8,16]	8	4	3
37	A2=[8,16,24]	10	5	1
38	A4=[24,32,40]	12	3	1

Moran's Test Results:
Table (2) shows the results of Moran's I test. The calculated Moran's I index equals −0.0449, with a standardized test statistic $Z = 8.0826$, which exceeds the critical value $Z_{0.025} = 1.96$ at the 5% significance level. Therefore, the null hypothesis of spatial randomness is rejected, and we conclude that the studied data exhibits statistically significant spatial dependence.

Table 2. Moran's Test Results

$\boldsymbol{I}$	$\boldsymbol{Z(I)}$
-0.0449	8.0826

Estimating Fuzzy Parameter Values for the FSLX Model:
The parameters for the Fuzzy Spatial Lag Explanatory (FSLX) model were estimated using the method of Symmetric Triangular ST-decomposition for the study data, as detailed in Table (3) below:

Table 3. FSLX Model Parameters - Fuzzy Lower Bound

$\hat{\boldsymbol{\beta}}_{0L}$	$\hat{\boldsymbol{\beta}}_{1L}$	$\hat{\boldsymbol{\beta}}_{2L}$	$\hat{\boldsymbol{\beta}}_{3L}$	$\hat{\boldsymbol{\beta}}_{4L}$	$\hat{\boldsymbol{\beta}}_{5L}$	$\hat{\boldsymbol{\beta}}_{6L}$
8.45	0.038	0.44	0.28	-0.49	0.7	0.27

$\hat{Y}L = (8.45) + (0.038)X_1 + (0.44)X_2 + (0.28)X_3 + (-0.49)WX_1 + (0.7)WX_2 + (0.27)WX_3$ … … … (20)

Table 4. FSLX Model Parameters - Fuzzy center Bound

$\hat{\boldsymbol{\beta}}_{0L}$	$\hat{\boldsymbol{\beta}}_{1L}$	$\hat{\boldsymbol{\beta}}_{2L}$	$\hat{\boldsymbol{\beta}}_{3L}$	$\hat{\boldsymbol{\beta}}_{4L}$	$\hat{\boldsymbol{\beta}}_{5L}$	$\hat{\boldsymbol{\beta}}_{6L}$
16.45	0.038	0.44	0.28	-0.49	0.7	0.27

$\hat{Y}C = (16.45) + (0.038)X_1 + (0.44)X_2 + (0.28)X_3 + (-0.49)WX_1 + (0.7)WX_2 + (0.27)WX_3$ … … … (21)

Table 5. FSLX Model Parameters - Fuzzy Upper Bound

$\hat{\boldsymbol{\beta}}_{0U}$	$\hat{\boldsymbol{\beta}}_{1U}$	$\hat{\boldsymbol{\beta}}_{2U}$	$\hat{\boldsymbol{\beta}}_{3U}$	$\hat{\boldsymbol{\beta}}_{4U}$	$\hat{\boldsymbol{\beta}}_{5U}$	$\hat{\boldsymbol{\beta}}_{6U}$
24.45	0.038	0.44	0.28	-0.49	0.7	0.27

$\hat{Y}U = (24.45) + (0.038)X_1 + (0.44)X_2 + (0.28)X_3 + (-0.49)WX_1 + (0.7)WX_2 + (0.27)WX_3$ … … … (21)

Discussion
The statistical results and estimated parameters for (FSLX) model estimated via equations (20), (21), and (22) using the method of ST-decomposition indicate that the intercept parameter $\hat{\beta}_0$ exhibits a monotonically increasing variation across the three bounds of the triangular fuzzy number. Specifically, its values were recorded as $\hat{\beta}_{0l} = 8.45$ at the lower bound $\hat{\beta}_{0c} = 16.45$ at the center bound, reaching $\hat{\beta}_{0u} = 24.45$ at the upper bound. This accurately reflects the fuzzy range and the underlying uncertainty surrounding the autonomous baseline of traffic accident fatalities. Regarding the original explanatory variables, a statistically significant positive effect is observed for the number of rollover accidents X_2 with an estimated parameter of 0.44, and pedestrian hit-and-run accidents X_3 with an effect size of 0.28; implying that an increase in these incidents directly elevates mortality rates. Conversely, collision accidents X_1displayed a marginally weak positive effect across all fuzzy levels, approaching zero 0.038. At the level of spatial spillovers originating from neighboring provinces, the spatially lagged variables revealed

a robust positive spatial impact for the second variable $WX_5$with an estimated parameter of 0.70, followed by the third variable $WX_6$with a positive effect of 0.27. In contrast, the first spatially lagged variable WX_4exerted a negative spatial spillover effect, with an estimated parameter of -0.49. The exact numerical symmetry (identity) of these parameter estimates across all three fuzzy levels for both original and spatially lagged variables demonstrates that the fuzziness and variance inherent in the response data Y are entirely absorbed by the model's intercept term. Consequently, this volatility does not compromise the stability or magnitude of the structural slopes. This confirms the efficiency of the applied estimation approach in isolating fuzzy noise while delivering a robust, highly stable analytical framework for spatial traffic accident modeling under conditions of imprecision and spatial heterogeneity.

Conclusion

Based on the fuzzy parameter estimates of (FSLX) model computed via the method of ST-decomposition, a positive relationship was established between the independent variables specifically rollover accidents X_2 and pedestrian hit-and-run accidents X_3 and the response variable, traffic accident fatalities Y. An increase in either predictor directly escalates the mortality count. Conversely, collision accidents X_1 demonstrated an exceptionally weak positive effect. Furthermore, spatial spillover effects from neighboring provinces varied; spatially lagged variables WX_5 and WX_6 exhibited positive spatial impacts, whereas WX_4 exerted a negative spatial spillover effect. The results of Moran's I test confirmed the statistical significance of spatial autocorrelation and spatial dependence between the explanatory variables (including spatial spillovers) and the fuzzy response variable (traffic accident fatalities across six Iraqi provinces in 2018). The calculated test statistic $I = 8.0826$ exceeded the critical value of 1.96, verifying the positive nature of the spatial association and providing statistical justification for spatial modeling due to the non-independence of geographical observations. The estimation approach proposed by ST-decomposition proves highly effective for fitting (FSLX) model. Its robust efficiency in yielding stable estimators across all three fuzzy bounds (lower, center, and upper) enables it to effectively absorb data volatility, thereby offering a reliable decision-support framework for traffic and safety policy planners. Conduct future research utilizing alternative parametric estimation techniques, such as Tanaka methods, to estimate (FSLX) model and benchmark their performance against the approach of ST-decomposition. Extend the implementation of ST-decomposition method to other spatial econometric and spatial lag model specifications. Assessing model stability via $\alpha - cut$ techniques to evaluate performance consistency under multiple methodologies Develop a fuzzy spatio -temporal model—subject to data availability—to capture dynamic spatial dependencies over time, thereby enhancing predictive capabilities and forecasting accuracy. Utilize larger datasets (exceeding N = 38) in future empirical applications to increase statistical power, refine the diagnostic sensitivity of spatial autocorrelation tests, and improve parameter estimation precision. Sensitivity to Spatial Weight Matrices: Test alternative spatial weight matrix (W) specifications—including topological contiguity, inverse distance, and (KNN) to assess the sensitivity and robustness of the empirical findings against variations in spatial structure.

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