

MATHEMATICAL MODELING OF THE HYDRODYNAMIC PROCESS OF IN-SITU LEACHING TAKING INTO ACCOUNT THE CHANGES HYDRODYNAMIC PARAMETERS IN A POROUS MEDIUM

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Abstract:

This scientific work examines the hydrodynamic processes pertinent to in-situ leaching, focusing on the extraction of precious metals from ore deposits through acid. A mathematical model, grounded in the principles of filtration-convection and diffusion characteristic of underground leaching, has been formulated to facilitate comprehensive analysis, monitoring, and prediction of system behavior. This model accounts for the influence of fluctuating hydrodynamic parameters, notably the filtration coefficient and medium porosity, which are contingent upon pressure levels and process kinetics. The main emphasis in this paper is on the variation of porosity as a function of pressure. Also, one of research objective is the protection of groundwater from potential contamination sources.

Keywords: In-situ leaching, mathematical modeling, minerals, filtration, diffusion, kinetics, useful component

Introduction

The identification, development, and optimal utilization of geological resources represent one of fundamental imperatives for all societies. Contemporary technological progress necessitates the extraction and application of subsurface mineral reserves. Consequently, research into efficient mineral extraction methodologies is of paramount importance [1]. The increasing adoption of in-situ leaching (ISL) as an advanced technique necessitates sophisticated mathematical modeling of the underlying subsurface processes. This, in turn, requires the development of increasingly complex mathematical models (MM) that demand advanced computational resources for their numerical solutions [2].

In an algorithm for finding a numerical solution to the model considered in this work was presented. The partial differential equation was reduced to a system of implicit equations using the given initial and boundary conditions and a finite difference method with a non-explicit form using first-order approximation with respect to time and using second-order approximation with respect to space variables. In this case, unlike other works, porosity is considered as a function of pressure [3].

The scientific work is devoted to the research, development and implementation of algorithms for solving MM and filtration-diffusion problems, which are an indispensable tool for conducting comprehensive research and making management decisions in order to improve the efficiency of mineral deposit development and groundwater protection during acid treatments.

In a two-dimensional unsteady hydrodynamic problem of underground leaching process is considered, the problem is accompanied by changes in the parameters of the solution and the productive water-bearing horizon along the (x,z) axes [4].

The hydrodynamic processes occurring in metal mines are studied in with particular attention to the method of underground mixing of precious metal ores, namely acid leaching. A mathematical model for analyzing, monitoring, and predicting the behavior of fluids in porous media has been developed, which incorporates filtration-convection and diffusion processes characteristic of underground flow. The model takes into account changes in the main hydrodynamic parameters, permeability and porosity, which are considered pressure-dependent and influenced by the process kinetics. Analysis shows that pressure changes caused by acid injection and mixing significantly affect the filtration properties of the ore layer [5].

Study investigates hydrodynamic process occurring in underground mining operations, with a focus on the acid in-situ leaching of precious metal ores. A mathematical model was developed to analyze, monitor, and predict fluid behavior within the ore body, incorporating filtration-convection and diffusion mechanisms typical of subsurface flow. A numerical solution algorithm was developed for the considered model and computational experiments were conducted based on this algorithm. The results of the computational experiments were compared and analyzed.

In-situ leaching, a diffusive process, involves the transfer of system components from the solid to the liquid phase. The driving force for this process is the concentration gradient between the solvent and leached component at the solid-liquid interface and the bulk liquid phase. In-situ leaching, as a filtration process, transfers the solvent from one point (from the injection well) to another and transports the dissolved products to the outlet (to the production well) together with the filter flow of the solvent. The ore-bearing rock is usually sand with an admixture of clays and other minerals. Useful components make up tenths or hundredths of a percent [6].

The initial stage of leaching, a kinetically driven process, involves a liquid-solid reaction progressing from the particle surface to its core. Mineral dissolution in acids fundamentally relies on hydrogen ion addition to the dissolved compound's anion; however, the overall dissolution rate is independent of this reaction rate, as such reactions are virtually instantaneous.

Physical statement of the problem

Based on the above, to study the underground leaching process, it is necessary to determine the concentration function of the useful component of the $C_2(x, y, z, t)$ in a limited heterogeneous region

$$G = \{(x, y, z, t), 0 < x < L_x, 0 < y < L_y, 0 < z < L_z, 0 < t \leq T\}.$$

In this case, the propagation of the pressure field H and the propagation of the porosity are determined from the equations of the elastic filtration regime

$$\beta h \frac{\partial(mH)}{\partial t} = \frac{\partial}{\partial x} \left[\kappa h \frac{\partial H}{\partial x} \right] + \frac{\partial}{\partial y} \left[\kappa h \frac{\partial H}{\partial y} \right] + \frac{\partial}{\partial z} \left[\kappa h \frac{\partial H}{\partial z} \right] + F_1 - F_2, \quad (1)$$

with initial

$$H|_{t=0} = H_0, \quad (2)$$

and boundary conditions

$$\kappa \frac{\partial H}{\partial x} \Big|_{x=0} = -l_1 \xi (H - H_0), \quad (3)$$

$$\kappa \frac{\partial H}{\partial x} \Big|_{x=L_x} = l_1 \xi (H - H_0), \quad (4)$$

$$\kappa \frac{\partial H}{\partial y} \Big|_{y=0} = -l_2 \xi (H - H_0), \quad (5)$$

$$\kappa \frac{\partial H}{\partial y} \Big|_{y=L_y} = l_2 \xi (H - H_0), \quad (6)$$

$$\frac{\partial H}{\partial z} \Big|_{z=0} = 0, \quad (7)$$

$$\kappa \frac{\partial H}{\partial z} \Big|_{z=L_z} = l_3 \xi (H - H_0), \quad (8)$$

The propagation of the reactant field is determined by solving the convective diffusion equation

$$\begin{aligned} \frac{\partial(mC_1)}{\partial t} = & \frac{\partial}{\partial x} D_{11} \left[\frac{\partial C_1}{\partial x} + \frac{\partial C_1}{\partial y} + \frac{\partial C_1}{\partial z} \right] + \frac{\partial}{\partial y} D_{22} \left[\frac{\partial C_1}{\partial x} + \frac{\partial C_1}{\partial y} + \frac{\partial C_1}{\partial z} \right] + \\ & + \frac{\partial}{\partial z} D_{33} \left[\frac{\partial C_1}{\partial x} + \frac{\partial C_1}{\partial y} + \frac{\partial C_1}{\partial z} \right] - \frac{\partial(V_x C_1)}{\partial x} - \frac{\partial(V_y C_1)}{\partial y} - \frac{\partial(V_z C_1)}{\partial z}, \end{aligned} \quad (5)$$

with initial

$$C_1|_{t=0} = 0, \quad (6)$$

and boundary

$$C_1 = 0, \quad (x, y, z) \in G_r. \quad (7)$$

The desired distribution of the concentration function of the useful component is determined by solving the following equation

$$\begin{aligned} \frac{\partial(mC_2 + N)}{\partial t} = & \frac{\partial}{\partial x} \left(D_{11} \left[\frac{\partial C_2}{\partial x} + \frac{\partial C_2}{\partial y} + \frac{\partial C_1}{\partial z} \right] \right) + \frac{\partial}{\partial y} D_{22} \left[\frac{\partial C_2}{\partial x} + \frac{\partial C_2}{\partial y} + \frac{\partial C_2}{\partial z} \right] + \\ & + \frac{\partial}{\partial z} D_{33} \left[\frac{\partial C_2}{\partial x} + \frac{\partial C_2}{\partial y} + \frac{\partial C_2}{\partial z} \right] - \frac{\partial(V_x C_2)}{\partial x} - \frac{\partial(V_y C_2)}{\partial y} - \frac{\partial(V_z C_2)}{\partial z}, \end{aligned} \quad (9)$$

with initial

$$C_2|_{t=0} = C_{2,0}, \quad (10)$$

and boundary

$$\left. \frac{\partial C_2}{\partial x} \right|_{x=0} = -\iota_1 \zeta (C_2 - C_{2,0}), \quad (30)$$

$$\left. \frac{\partial C_2}{\partial x} \right|_{x=L_x} = \iota_1 \zeta (C_2 - C_{2,0}), \quad (31)$$

$$\left. \frac{\partial C_2}{\partial y} \right|_{y=0} = -\iota_2 \zeta (C_2 - C_{2,0}), \quad (32)$$

$$\left. \frac{\partial C_2}{\partial y} \right|_{y=L_y} = \iota_2 \zeta (C_2 - C_{2,0}), \quad (33)$$

$$\left. \frac{\partial C_2}{\partial z} \right|_{z=0} = 0, \quad (34)$$

$$\left. \frac{\partial C_2}{\partial z} \right|_{z=L_z} = \iota_3 \zeta (C_2 - C_{2,0}). \quad (35)$$

The equation of mass exchange kinetics, which determines the rate of transition of a substance from one phase to another, has the following form:

$$\begin{aligned} \frac{\partial N}{\partial t} &= \gamma(C_1) f(C_2, N, t), \\ N|_{t=0} &= N^0. \end{aligned} \quad (13)$$

The rate of change in these hydrodynamic parameters is pressure-dependent; exhibiting exponential behavior under high pressure and linear behavior under low pressure conditions:

$$\begin{aligned} m &= m_0 e^{-\beta_c (H_0 - H)/m_0} \\ m &= m_0 + \beta_c (H - H_0) \end{aligned}$$

It should be noted that during the underground leaching process, due to the effect of the reagent on the ore deposits, a chemical reaction occurs and the substance passes from one phase to another, as a

result of which the hydrodynamic parameters of the porous medium (filtration and porosity coefficients) and the pressure in the ore collector change [7].

H – pressure value, (m); H_0 – initial pressure value, (m); m – value of the porosity coefficient; γ – bulk density of the solution, (kg/m^3); k – filtration coefficient, (m/day); t – time, h – thickness of the ore horizon (m); β – elastic capacity coefficient, (sm^2/kg); l_1, l_2, l_3 – constants taking the values 0 or 1; ζ

– coefficient of non-dimensionalization of equations (1/day);

$$F_1 = \sum_{i,j=1}^{N_1} q_{1,i}(t) \delta(x - x_{i,j}, y - y_{i,j}, z - z_{i,j})$$

$$F_2 = \sum_{i,j=1}^{N_2} q_{2,j}(t) \delta(x - x_{i,j}, y - y_{i,j}, z - z_{i,j})$$

$$\delta_{i,j} = \begin{cases} 1, & x = x_{i,j}, y = y_{i,j}, z = z_{i,j}, \\ 0, & \text{all others conditions} \end{cases}$$

 $q_{i,j}(t)$ is well flow rates;

– this is the Dirac delta function; C_1 – concentration of injected liquid; C_2 – concentration of pregnant solution; D_{11}, D_{22}, D_{33} – coefficient of diffusion; V_x, V_y, V_z – filtration velocities, they are

determined by Darcy's law:

$$V_x = -k \frac{\partial H}{\partial x}, V_y = -k \frac{\partial H}{\partial y}, V_z = -k \frac{\partial H}{\partial z};$$

 G_r – layer boundary; ζ – coefficient of non-dimensionalization of equations (1/m); N – kinetic of process; γ – bulk density of the solution (kg/m^3); m_0 – porosity coefficient at $H = H_0$.

Equations of MM are parabolic type linear partial differential equations with the given conditions, finding them solutions analytically are very difficult problem [8]. The finite difference method was used to solve this equation numerically, and other words that solving a finite difference scheme with a non-explicit form using first-order approximation with respect to time and space variables. According to this reason, the numerical solution of the problem was solved in work of the authors.

Materials and Methods

The study follows the mathematical modeling and computational experiment approach, which was carried out in four stages: (i) formulation of a physical and mathematical model of the in-situ leaching process; (ii) discretization of the model equations by the finite difference method; (iii) implementation of the numerical algorithm in software developed by the authors; and (iv) analysis of the computational results for pressure, porosity, filtration velocity and concentration fields.

The mathematical model consists of three coupled blocks, which are described in the previous section. The first block describes the pressure field in the ore-bearing horizon under the elastic filtration regime, where the filtration velocities are obtained from Darcy's law. The second block is the convective diffusion equation for the reagent (acid) concentration. The third block is the convective diffusion equation for the concentration of the useful component, which includes the mass exchange kinetics between the solid and liquid phases. The filtration coefficient and the porosity are not constant: they are treated as pressure-dependent functions, which change linearly at low pressure and exponentially at high pressure, so that the hydrodynamic and transport blocks influence each other.

The model equations are linear parabolic partial differential equations with given initial and boundary conditions, for which analytical solutions are difficult to obtain. Therefore, they were solved numerically using an implicit (non-explicit) finite difference scheme with first-order approximation in time and second-order approximation in the space variables. The implicit form was chosen because of its stability properties, and it reduces the differential problem to a system of algebraic equations on each time layer. The detailed derivation of the scheme and the solution algorithm is given in.

The computation is organized sequentially on each time layer. First, the pressure field is calculated using the current values of the filtration coefficient and porosity. Second, the porosity and the filtration coefficient are updated according to the new pressure values. Third, the filtration velocities are determined from Darcy's law. Finally, the reagent concentration and the concentration of the useful component are calculated, and the calculation proceeds to the next time layer.

The computational experiments were performed for a rectangular ore-bearing area of $100 \text{ m} \times 50 \text{ m}$, where the thickness of the layer (the z -coordinate) varies depending on the ore reserve. The well pattern consists of one production well located in the center of the area and two injection wells located symmetrically on both sides of it. The process was simulated for 15 days, and the results are reported for days 1, 5, 10 and 15.

Results

The calculations were carried out on the basis of a software developed by the authors of the article. These capabilities are essential for simulating fluid flow and mass transport within ore-bearing formations. When solving the problem, the dimensions of the sphere were taken as follows: $0 < x < 100 \text{ (m)}$, $0 < y < 50 \text{ (m)}$, z -varies depending on the ore reserve. **Fig. 1–3** show the distribution of acid in the ore-bearing plain within the mine [9]. Production well is located in the center, and two injection wells are located at equal distances on both sides of it. Over time, the pressure increases and the dynamics of acid distribution along the sphere expands.

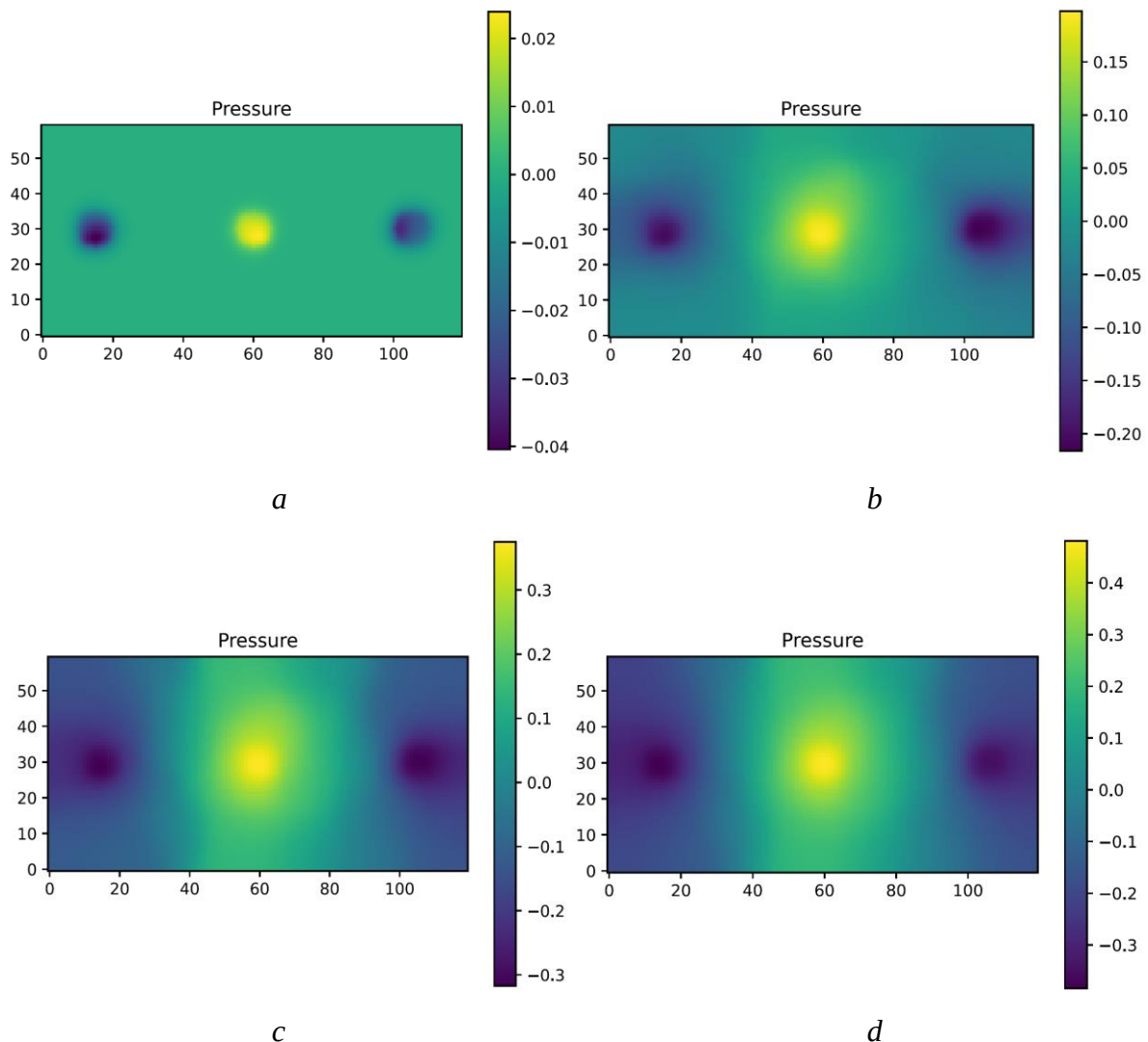


Fig1. The main signature: *a* – pressure distribution in the area at day 1; *b* – pressure distribution in the area at day 5; *c* – pressure distribution in the area at day 10; *d* – pressure distribution in the area at day 15.

In Figure 1, a medium with an ore layer is located, injection well is located in the center of the area, and production wells are located at the 2 sides. As a result of the injection of acid from the injection well through the pressure, the pressure spreads throughout the area. It can be seen on graph a that on the first day of the process, the pressure spreads starting from the injection well. Graph b shows by day 5, the pressure distribution radius has lengthened and the pressure value around the production wells has decreased [10]. Graph c illustrates by day 10, the pressure distribution is approaching an equilibrium state. In graph d, the pressure distribution is balanced between the injection and production wells by day 15

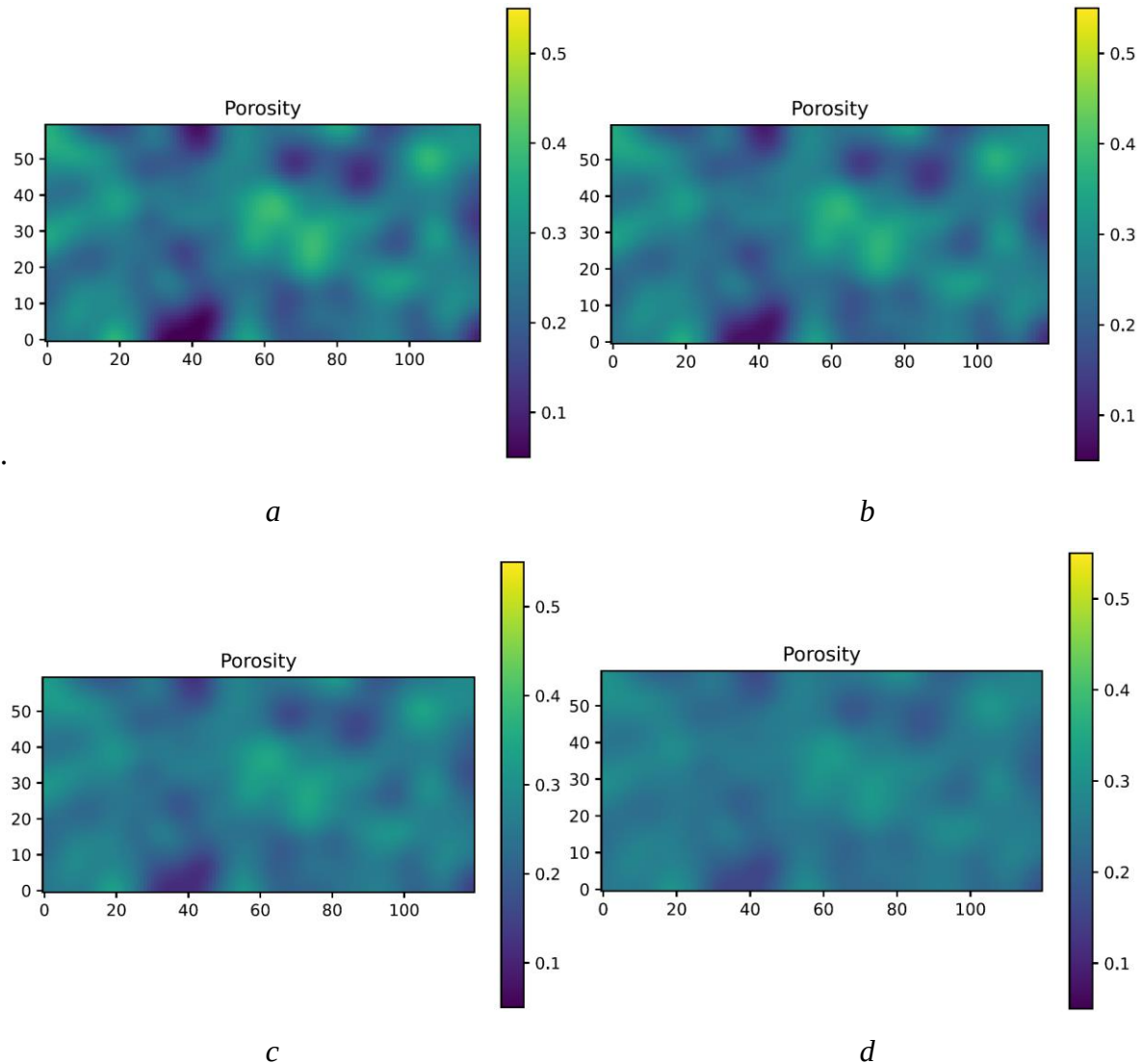


Fig2. The main signature: *a* – porosity of the medium at day 1; *b* – porosity of the medium at day 5; *c* – porosity of the medium at day 10; *d* – porosity of the medium at day 15.

Figure 2 shows the dynamics of the change in porosity over time in the medium [11]. The porosity on day 1 of the process is shown in graph *a*. Graph *b* shows the porosity on day 5 of the process, which shows that the porosity increases with time. Graph *c* shows the porosity on day 10, showing that the porosity of the medium has increased compared to the initial experiments. Graph *d* shows the porosity of the medium on day 15, showing that the porosity has almost reached a state of equilibrium in all parts of the medium [12].

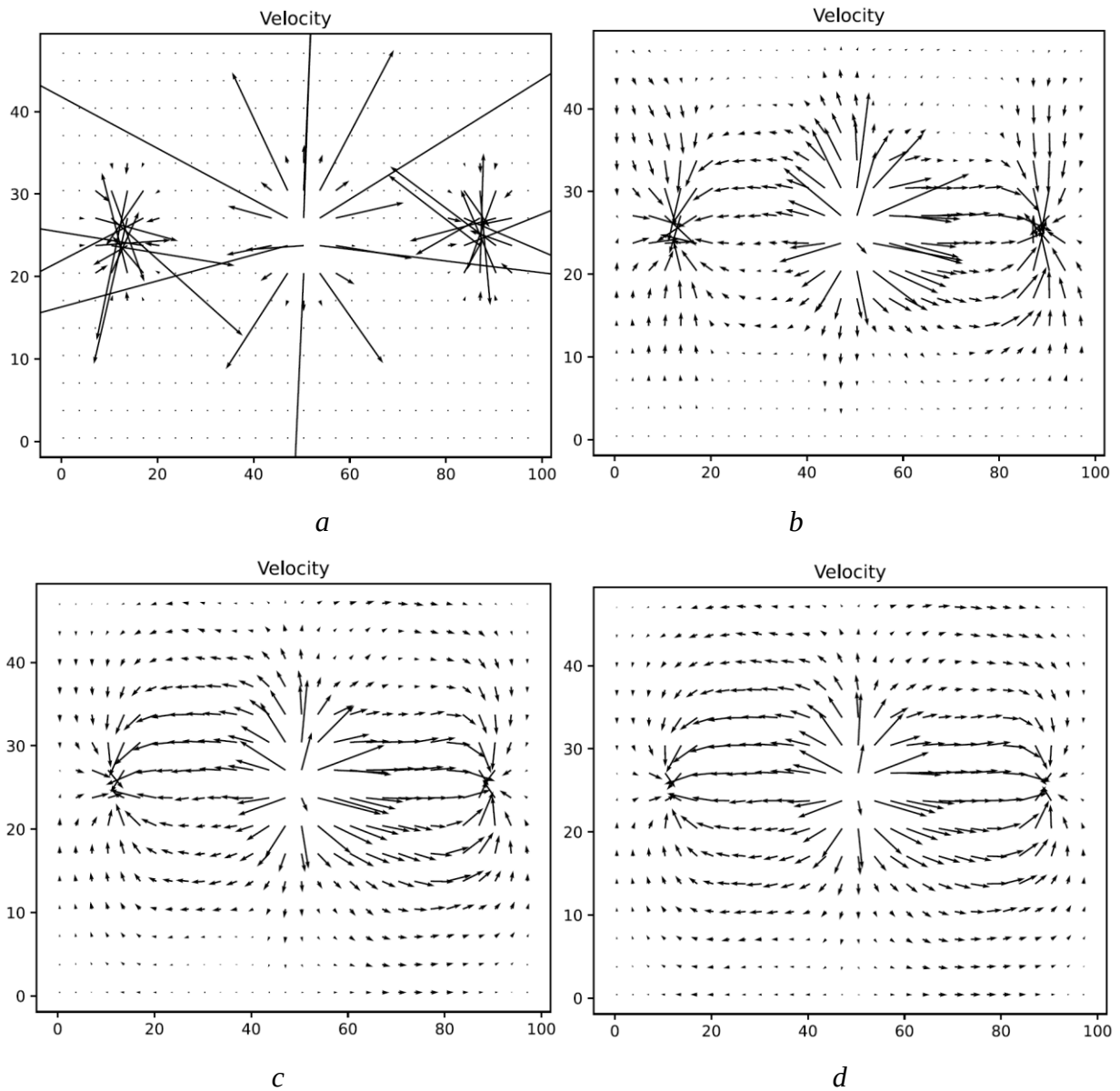


Fig3. The main signature: *a* – velocity at day 1; *b* – velocity at day 5; *c* – velocity at day 10; *d* – velocity at day 15.

Discussion

Figure 3 shows the flow rates of the liquid and the mixture sent through the injection well. The flow from the center is reflected in the movement towards the production wells. Graph a shows the flow rate at the beginning of the process and its magnitude. Graph b shows that by day 5 the speed has slowed down and the directions have increased due to the increase in porosity. Graph c and graph d show the flow rates on the 10th and 15th days of the process[13]. It can be seen that over time the speed value decreases and the liquid finds directions for itself [14].

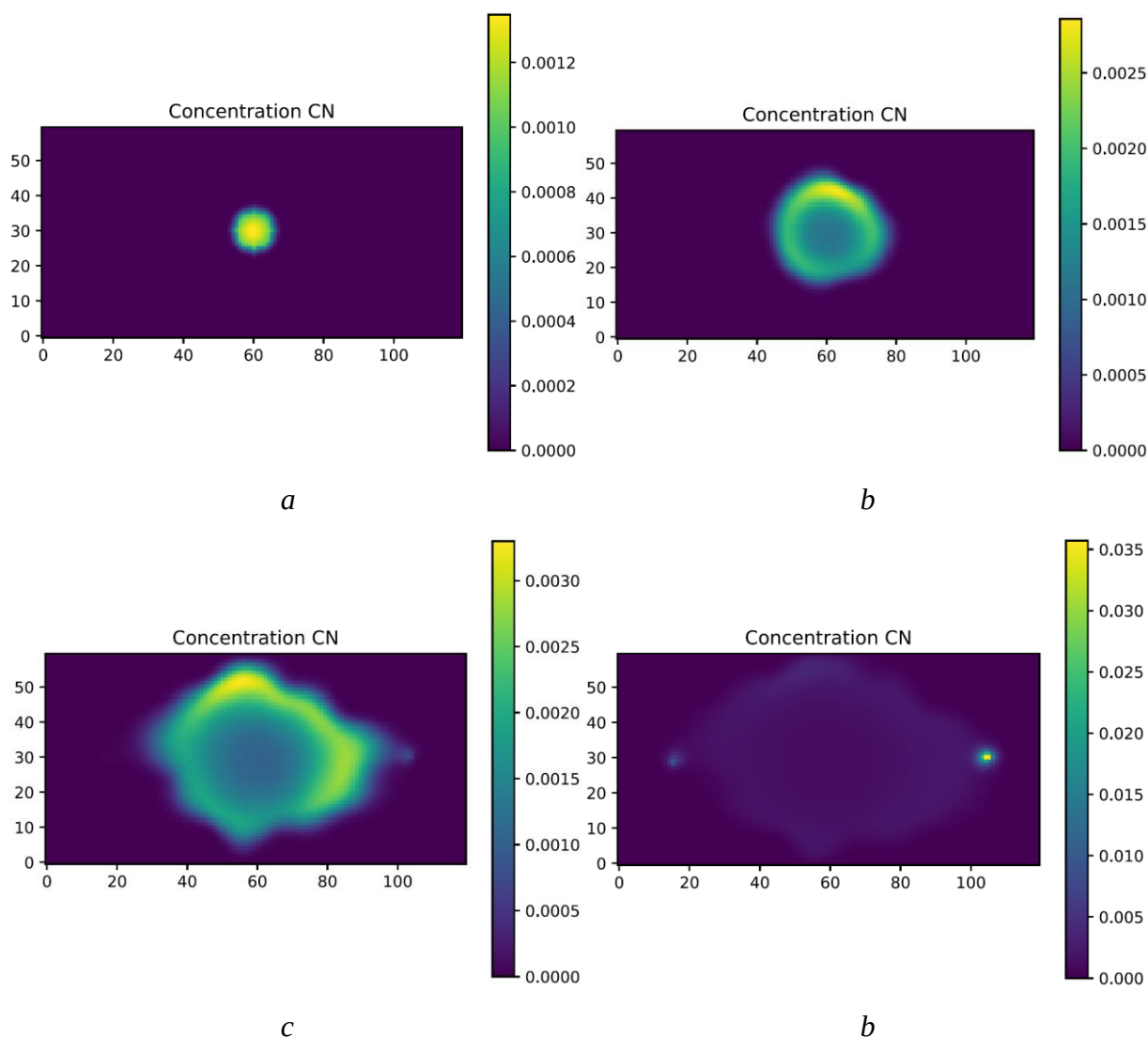


Fig4. The main signature: *a* – distribution of concentration at day 1; *b* – distribution of concentration at day 5; *c* – distribution of concentration at day 10; *d* – distribution of concentration at day 15.

The distribution of acid injected from the injection well is shown in Figure 4. Under pressure, the acid diffuses into the porous medium and mixes with the ores to form a liquid mixture. Graph *a* shows the situation on the day the acid injection process from the source well began. Graph *b* shows the state of acid diffusion by the 5th day of the process, the radius of acid diffusion has reached almost 20 m. Graph *c* shows that by the 10th day of the process, the mixture spreads faster in areas with high porosity and slower in areas with low porosity, and this mixture has almost reached the production wells [15]. Graph *d* shows that by the 15th day of the In-situ leaching process, the mixture has completely reached the production well, and the mixture has reached a state of equilibrium between the injection and production wells

Conclusion

ISL is a complicated process which involves injecting solutions into subsoil with data regarding the progression of pregnant solution available only at wells. True picture cannot be observed but can be simulated with appropriate physicochemical models by using which, optimal well pattern can be determined to increase profitability of production, as demonstrated in current work. The main parameters of this study were pressure, porosity, flow rate and concentration. Considering porosity as a function of pressure allows for a more complete understanding of the process and contributes to the effectiveness of the results obtained. The results show that after the porosity increases, the process is stopped soon and the amount of useful metal in the mixture decreases. This means that this method allows for the rapid recovery of minerals. Analysis of the problem statement reveals that variations in

pressure within the ore deposit, induced by solution injection and extraction, directly impact the permeability and porosity coefficients of the stratum.

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