

EQUILIBRIUM OF YARNS ON THE SURFACE OF A 2/1 TWILL FABRIC

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Abstract:

This study is devoted to the theoretical and practical investigation of the equilibrium state of warp and weft yarns on the surface of a 2/1 twill woven fabric. The equilibrium arrangement of yarns within the fabric structure is one of the key factors determining the geometric parameters, surface topography, mechanical properties, and service performance of woven materials. Therefore, determining the equilibrium state of yarns formed under the influence of yarn tension, bending, mutual compression, and interaction forces at interlacing points during the weaving process is an important scientific challenge in modern textile engineering.

The main objective of this research is to identify the regularities governing the arrangement of yarns on the surface of a 2/1 twill fabric, evaluate the structural and technological factors affecting their equilibrium state, and describe the geometric configuration of the yarns using mathematical modeling approaches. The study comprehensively analyzes fabric geometry, yarn bending radius, yarn crimp, interaction forces at interlacing points, tension distribution, and the spatial configuration of warp and weft yarns. The investigation is based on theoretical analysis, mathematical modeling, and structural analysis of woven fabrics.

The obtained results demonstrate that the equilibrium state of yarns in a 2/1 twill fabric significantly influences the surface topography, fabric thickness, areal density, air permeability, deformation behavior, and mechanical strength of the material. Furthermore, maintaining an optimal ratio between warp and weft yarn tensions ensures a more uniform yarn distribution over the fabric surface, reduces contact stresses, and improves both the appearance and functional performance of the fabric. Conversely, disturbance of yarn equilibrium leads to surface irregularities, localized deformation, increased mechanical stresses, and deterioration of fabric quality.

The findings of this research have considerable scientific and practical significance for the structural design of twill woven fabrics, optimization of weaving process parameters, improvement of mathematical models describing fabric structure, and development of high-quality textile materials. The proposed approaches can be applied in selecting weaving machine operating conditions, controlling yarn tension, and predicting the quality characteristics of finished woven fabrics.

Keywords: 2/1 twill woven fabric; yarn equilibrium; geometric model of the fabric in the warp and weft directions; warp and weft yarn crimp height; yarn deformation and compression.

Introduction

The performance, deformation, and aesthetic properties of fabrics are directly associated with the spatial arrangement of yarns within the fabric structure and their state of mechanical equilibrium. In particular, in twill-woven fabrics, the systematic arrangement of yarns on the fabric surface forms a characteristic diagonal texture, which significantly influences the surface geometry, internal stress distribution, and abrasion resistance of the fabric. From this perspective, the theoretical and geometrical investigation of the equilibrium state of yarns in a 2/1 twill fabric represents one of the key stages in the scientific analysis of fabric structure [1].

In a 2/1 twill fabric, the warp and weft yarns intersect periodically, with each yarn floating over the fabric surface in certain regions while passing beneath the yarns of the opposite system in others. Consequently, the yarns are not arranged along straight lines but follow curved three-dimensional trajectories. As a result, internal bending moments, normal contact forces, tensile forces, and frictional forces are generated within the yarns. When these forces reach a state of equilibrium, the fabric structure attains a stable mechanical configuration [2].

In a 2/1 twill weave, the equilibrium of yarns is primarily determined by their geometric parameters, including yarn diameter, crimp wave height, repeat length, spacing between adjacent yarns, and structural density. In addition, the physical and mechanical properties of the yarns, particularly the elastic modulus, bending rigidity, and coefficient of friction, significantly influence the distribution of forces within the fabric structure. As a result, a balanced three-dimensional configuration of the warp and weft yarns is established within the fabric [3].

In 2/1 twill fabrics, the longer surface floats of the yarns alter their bending curvature and create a characteristic microrelief on the fabric surface. This structural feature directly affects the functional properties of the fabric, including softness, drapeability, air permeability, and abrasion resistance. In particular, for suiting fabrics, the balanced spatial arrangement of warp and weft yarns plays a crucial role in maintaining the dimensional stability of the fabric appearance, surface smoothness, and resistance to deformation [4].

Methodology. Therefore, the analysis of the equilibrium state of yarns in a 2/1 twill fabric based on geometric and mechanical models makes it possible to determine the actual yarn trajectories, the degree of yarn crimp, contact forces, the distribution of internal stresses, and the potential energy state of the fabric structure. Such an approach provides a deeper understanding of the interaction mechanisms between warp and weft yarns, establishes the theoretical principles governing fabric structure formation, and enables the scientific optimization of technological parameters during the weaving process. Furthermore, by optimizing the force equilibrium between the warp and weft yarn systems, it becomes possible to stabilize the surface relief of the fabric, reduce contact pressures, improve abrasion resistance, minimize residual deformations, and enhance both the aesthetic appearance and the service performance of the final product. The theoretical findings obtained provide a fundamental scientific basis for the design of high-quality suiting fabrics, the prediction of their physical and mechanical properties, and the further improvement of weaving technology.

In a 2/1 twill fabric, the weft yarn alternately interlaces with the warp yarns according to the specified weave repeat, appearing on the fabric surface in certain regions while passing beneath the warp yarns in others. Consequently, the geometric axis of the weft yarn is not a straight line but a periodically varying three-dimensional curved trajectory. The formation of this trajectory is governed by the elastic properties of the yarn, its tension level, cross-sectional geometry, the packing density of the warp yarns, and the contact pressures developed between adjacent yarns. At each interlacing point, the yarns undergo partial compression and deformation due to mutual contact pressure, causing the actual yarn geometry to deviate to some extent from the idealized theoretical model. Therefore, an accurate description of the weft yarn trajectory requires consideration not only of geometric parameters but also of the physical and mechanical properties of the yarn material, including bending rigidity, contact deformation, and frictional forces.

Result and Discussion. Such a three-dimensional arrangement of yarns within the fabric structure directly influences the formation of the surface relief, yarn crimp height, the degree of yarn waviness, deformation behavior under mechanical loading, and the overall mechanical

performance of the fabric. Consequently, mathematical modeling of the spatial trajectories of both warp and weft yarns constitutes one of the key scientific approaches for the comprehensive analysis of fabric structure, the optimization of structural parameters, and the development of high-quality woven fabrics [5].

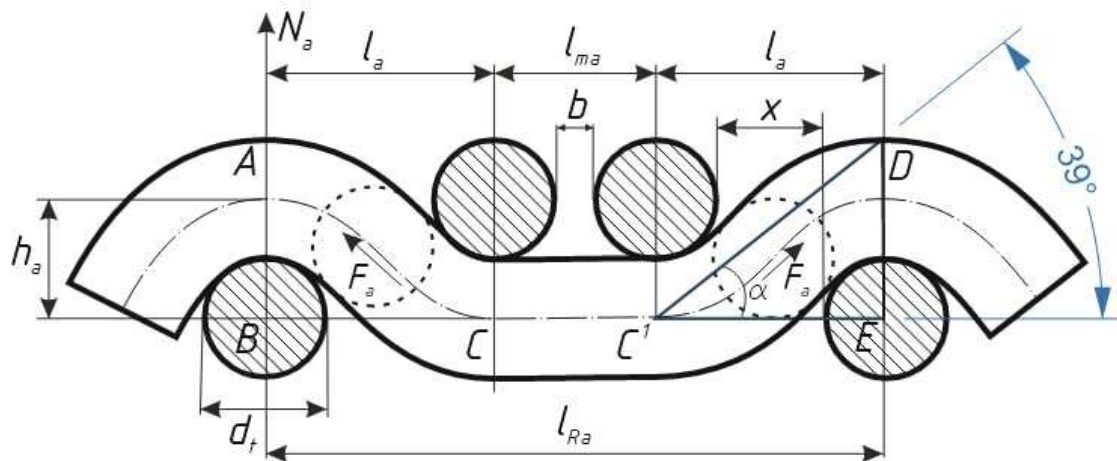


Figure 1. Geometric model of a balanced-surface 2/1 twill suiting fabric in the weft direction.

The total projected length of the weft yarn within one weave repeat is expressed as follows:

$$l_{Rt} = \frac{R_t \cdot 10}{P_a} = \frac{3 \cdot 10}{26,8} = 1,12 \text{ mm.}$$

$$l_{Ra} = \frac{R_a \cdot 10}{P_t} = \frac{3 \cdot 10}{25} = 1,2 \text{ mm.}$$

$$bt = (l_{Ro} - 5d = 1,12 - 1,1 = 0,02 \text{ mm;}$$

$$ba = (l_{Ra} - 5d = 1,2 - 1,1 = 0,1 \text{ mm;}$$

$$L_{Ry} = 2l_y + l_a = 0,88 + 0,32 = 1,2 \text{ mm}$$

$$L_{Rt} = 2l_y + l_t = 0,88 + 0,24 = 1,12 \text{ mm}$$

where:

l_a - the horizontal projection of the curved surface segment of the weft yarn;

l_t - the central straight segment between the centers of two adjacent warp yarns.

$$l_a = d + ba = 0,22 + 0,1 = 0,32 ; l_t = d + bt = 0,22 + 0,02 = 0,24$$

L_{Ry} - the weave repeat length in the weft direction [6].

The actual length of the weft yarn, taking into account the yarn crimp (crimp height), is expressed as follows:

$$S_a = 2\sqrt{l_a^2 + h_a^2} + l_a = 2\sqrt{0,44 + 0,275^2} + 0,32 = 1,14$$

where: h_y - the crimp wave height of the weft yarn.

If the yarn diameter is taken into account, the crimp height can be estimated using the following geometric relationship:

$$h_a = \frac{d_t + d_a}{2} + \Delta_a = \frac{0,22 + 0,22}{2} + 0,055 = 0,275 \text{ mm}$$

$$h_t = \frac{d_t + d_a}{2} + \Delta_t = \frac{0,22 + 0,22}{2} + 0,055 = 0,275 \text{ mm}$$

where: d_t - the diameter of the warp yarn; d_a - the diameter of the weft yarn;

Δ_a - the additional deflection of the weft yarn caused by yarn deformation, compression, and structural compaction within the fabric [7].

The value of Δ_a is usually expressed as a fraction of the yarn diameter:

$$0,15d \div 0,30d$$

is taken as a fraction of the yarn diameter.

In this case, assuming: $d \approx 0,22 \text{ mm}$ we obtain:

$$\Delta a \approx 0,22 \times 0,25 \approx 0,055 \text{ mm}$$

we obtain:

The crimp (shrinkage) coefficient of the weft yarn within the fabric structure is expressed as follows:

$$\epsilon_y = \frac{S_y - L_{Ry}}{L_{Ry}} \cdot 100\% = \frac{1,14 - 1,12}{1,12} \cdot 100\% = 1,8\%$$

or:

$$\epsilon_0 = \frac{2\sqrt{l_0^2 + h_0^2} + l_a - (2l_0 + l_a)}{2l_0 + l_1} 100\%$$

This equation demonstrates the degree of bending of the weft yarn in a 2/1 twill weave and indicates the additional length generated within the fabric structure due to yarn crimp [8].

Equilibrium equation of the weft yarn

In the curved sections, the weft yarn is in contact with the warp yarns. At the contact points, the yarn is subjected to normal pressure forces, friction forces, and tensile forces.

The elementary equilibrium condition of the weft yarn can be expressed as follows:

$$\sum F_x = 0, \sum F_y = 0$$

In the horizontal direction:

$$T_{y1} \cos \alpha_y = T_{y2} \cos \beta_y + F_y$$

In the vertical direction:

$$N_0 + T_{01} \sin \alpha_0 - T_{02} \sin \beta_0 = 0$$

where: T_{y1}, T_{y2} , T_{01} , T_{02} - the tensile forces acting on both sides of the weft and warp yarns, respectively; $\alpha_y, \beta_y, \alpha_0, \beta_0$ - the bending angles of the weft yarn [9]; N_y, N_0 - the normal reaction forces exerted by the warp yarn on the weft yarn; F_y, F_0 - the friction forces between the weft yarns or at the warp–weft contact points.

The bending angle can be determined geometrically as follows:

$$\tan \alpha_y = \frac{h_y}{l_y}$$

From this:

$$\alpha_y = \arctan \left(\frac{h_y}{l_y} \right)$$

The friction force:

$$F_y = \mu_y N_y$$

where: μ_y - the coefficient of friction in the weft direction.

Bending moment in the weft direction [10].

The bending moment of the weft yarn is determined as follows:

$$M_y = E_y I_y k_y$$

where: E_y - the elastic modulus of the weft yarn;

I_y - the moment of inertia of the weft yarn cross-section;

k_y - the curvature of the weft yarn.

Assuming that the yarn cross-section is cylindrical:

$$I_y = \frac{\pi d_y^4}{64}$$

The curvature can be simplified as:

$$k_y \approx \frac{2h_y}{l_y^2}$$

Therefore:

$$M_y = E_y \frac{\pi d_y^4}{64} \cdot \frac{2h_y}{l_y^2}$$

This equation is used to evaluate the bending resistance of the weft yarn.

Geometric model in the warp direction:

In a balanced-surface 2/1 twill suiting fabric, the warp yarn also follows a curved trajectory along the warp direction [11]. In this model, the horizontal projection length of the warp yarn within one weave repeat is expressed as follows:

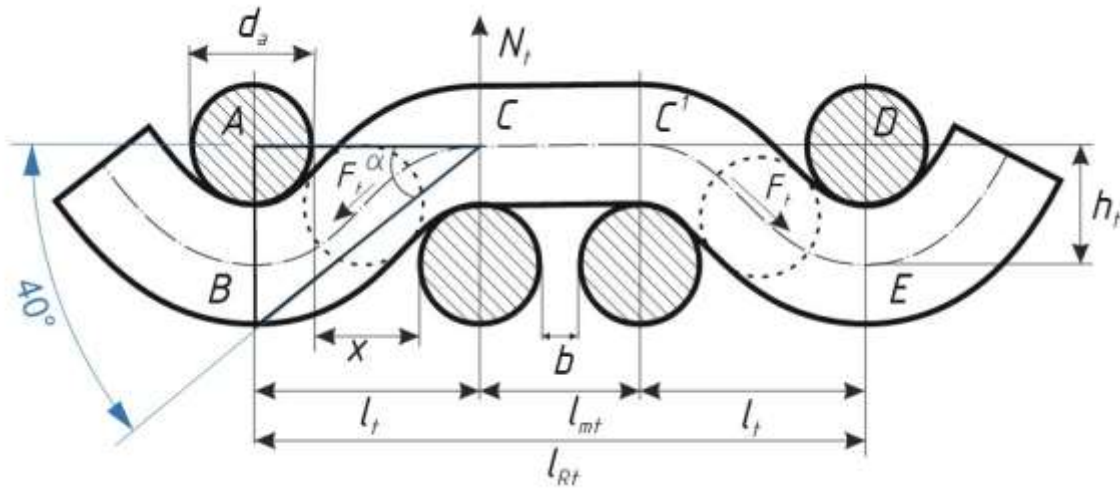


Figure 2. Geometric model of a balanced-surface 2/1 twill suiting fabric in the warp direction.

$$L_{Rt} = 2l_t + l_1$$

where: l_t - the horizontal projection of the curved surface segment of the warp yarn; l_1 - the central straight segment; L_{Rt} - the weave repeat length in the warp direction.

The actual length of the warp yarn is determined as:

$$S_t = 2\sqrt{l_t^2 + h_t^2} + l_1$$

where: h_t - the crimp wave height of the warp yarn.

The shrinkage (crimp) coefficient of the warp yarn within the fabric structure is expressed as:

$$\varepsilon_t = \frac{S_t - L_{Rt}}{L_{Rt}} \cdot 100\%$$

or:

$$\varepsilon_t = \frac{2\sqrt{l_t^2 + h_t^2} + l_1 - (2l_t + l_1)}{2l_t + l_1} \cdot 100\%$$

This expression characterizes the three-dimensional bending of the warp yarn in a 2/1 twill weave and describes the additional yarn length formed within the internal structure of the fabric [12].

Equilibrium equation of the warp yarn:

$$\sum F_x = 0, \quad \sum F_y = 0$$

In the horizontal direction:

$$T_{t1} \cos \alpha_t = T_{t2} \cos \beta_t + F_t$$

In the vertical direction:

$$N_t + T_{t1} \sin \alpha_t - T_{t2} \sin \beta_t = 0$$

where: T_{t1}, T_{t2} - the tensile forces acting on both sides of the warp yarn;

α_t, β_t - the bending angles of the warp yarn;

N_t - the normal reaction force at the contact point between the warp and weft yarns; F_t - the friction force acting in the warp direction.

The bending angle:

$$\tan \alpha_t = \frac{h_t}{l_t}$$

$$\alpha_t = \arctan\left(\frac{h_t}{l_t}\right)$$

The friction force:

$$F_t = \mu_t N_t$$

where: μ_t - the coefficient of friction in the warp direction.

Bending moment in the warp direction

The bending moment of the warp yarn is determined as:

$$M_t = E_t I_t k_t$$

where: E_t - the elastic modulus of the warp yarn; I_t - the moment of inertia of the warp yarn cross-section; k_t - the curvature of the warp yarn, which represents the degree of bending formed as a result of the warp yarn passing over and under the weft yarns rather than following a straight-line path within the fabric structure [13].

From a scientific perspective, the curvature of the warp yarn is related to the bending radius of the yarn in the fabric cross-section and can be explained as follows:

$$k_t = \frac{1}{R_t}$$

where: k_t - the curvature (bending) of the warp yarn, mm or 1/m;

R_t - the bending radius of the warp yarn, mm.

R_t - is large, the yarn undergoes less bending, and k_t - has a smaller value. If R_t is small, the yarn is bent more significantly, resulting in a higher value of k_t .

In a 2/1 twill fabric, the warp yarn alternately interlaces with the weft yarns, forming a specific crimp wave on the fabric surface. Therefore, k_t - is a geometric-mechanical parameter used to evaluate the degree of bending of the warp yarn within the fabric structure and its influence on yarn tension, deformation behavior, and equilibrium state [14].

$$I_t = \frac{\pi d_t^4}{64}$$

Curvature:

$$k_t \approx \frac{2h_t}{l_t^2}$$

Therefore, the bending moment of the warp yarn is:

$$M_t = E_t \frac{\pi d_t^4}{64} \cdot \frac{2h_t}{l_t^2}$$

Equilibrium condition for a balanced-surface 2/1 twill fabric

In a balanced-surface suiting fabric, the surface contributions of the warp and weft yarns are relatively balanced. Therefore, the geometric equilibrium condition can be expressed as:

$$h_a \approx h_t; S_a \approx S_t; \varepsilon_a \approx \varepsilon_t$$

These conditions indicate that the warp and weft yarns form a relatively equal surface relief on the fabric surface. The mechanical equilibrium condition is expressed as:

$$T_a \sin \alpha_a + T_t \sin \alpha_t = N$$

or, in a generalized form:

$$N = T_a \frac{h_a}{\sqrt{l_a^2 + h_a^2}} + T_t \frac{h_t}{\sqrt{l_t^2 + h_t^2}}$$

where: T_a - the tension force of the weft yarn; T_t - the tension force of the warp yarn; N - the total normal pressure between the warp and weft yarns.

Generalized energy equation for a 2/1 twill fabric

The stability of the fabric structure is evaluated based on the principle of minimum energy:

$$\Pi = U_a + U_t - W$$

where: U_a - the bending energy of the weft yarn; U_t - the bending energy of the warp yarn; W - the work performed by external forces..

The bending energy of the weft yarn is expressed as:

$$U_a = \frac{1}{2} \int_0^{L_{Ra}} E_a I_a k_a^2 dx$$

The bending energy of the warp yarn is expressed as:

$$U_t = \frac{1}{2} \int_0^{L_{Rt}} E_t I_t k_t^2 dx$$

The condition of stable equilibrium:

$$\delta \Pi = 0 \quad \text{i.e., } \delta(U_a + U_t - W) = 0$$

This equation provides a theoretical basis for describing the spatial arrangement, bending behavior, and equilibrium of contact forces between yarns in a balanced-surface 2/1 twill suiting fabric.

The two proposed geometric models demonstrate that, in a balanced-surface 2/1 twill suiting fabric, the warp and weft yarns are arranged along complex curved trajectories relative to each other. The actual length of the yarn in the weft direction is S_y , while in the warp direction it is S_0 . Their increase relative to the weave repeat projection determines the yarn crimp and internal bending deformation within the fabric structure [15].

Under the condition of surface balance:

$$\varepsilon_a \approx \varepsilon_t \quad \text{and} \quad h_a \approx h_t$$

the surface relief formed by the warp and weft yarns is considered to be in mutual equilibrium. This condition plays an important role in ensuring the dimensional stability of appearance, softness, drapeability, and service durability of suiting fabrics.

Therefore, the geometric and mechanical model of a balanced-surface 2/1 twill suiting fabric can be represented by the following system of fundamental equations:

$$\begin{aligned} L_{Ra} &= 2l_a + l_1 \\ S_a &= 2\sqrt{l_a^2 + h_a^2} + l_1 \\ \varepsilon_a &= \frac{S_a - L_{Ra}}{L_{Ra}} \cdot 100\% \\ L_{Rt} &= 2l_t + l_1 \\ S_t &= 2\sqrt{l_t^2 + h_t^2} + l_1 \\ \varepsilon_t &= \frac{S_t - L_{Rt}}{L_{Rt}} \cdot 100\% \end{aligned}$$

$$F_a = \mu_a N_a; \quad F_t = \mu_t N_t$$

$$\begin{aligned} M_a &= E_a \frac{\pi d_a^4}{64} \cdot \frac{2h_a}{l_a^2} \\ M_t &= E_t \frac{\pi d_t^4}{64} \cdot \frac{2h_t}{l_t^2} \end{aligned}$$

These equations serve as a scientific foundation for describing the geometric arrangement of yarns, crimp wave formation, yarn shrinkage, contact pressure, frictional forces, and mechanical equilibrium in a balanced-surface 2/1 twill suiting fabric.

Conclusion. In conclusion, the analysis of the equilibrium state of yarns in a 2/1 twill fabric based on geometric and mechanical models makes it possible to determine their spatial trajectories, bending characteristics, and interaction mechanisms. This approach provides an important scientific basis for optimizing fabric structures and designing suiting fabrics with improved physical, mechanical, and performance properties.

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